SYNCHRONIZATION OF LAI-CHEN (2016) CHAOTIC SYSTEM WITH ACTIVE CONTROL

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Abstract

Most of the events in the real world show non-linear behavior. Such events are usually chaotic. Chaotic systems are highly sensitive to the initial conditions and parameters values, exhibit non-periodic properties, and some have a very broad frequency spectrum. Because of these features, chaotic systems are used in different branches of science such as encryption, communication, random number generators, prediction algorithms, computer games, biology, medicine. In this regard, a variety of chaotic and hyper-chaotic systems are introduced in the literature. However, because of chaotic systems are very sensitive to initial conditions and parameters, chaotic systems need to be synchronized in order to be used in chaos-based communication and encryption applications. In this study, a new chaotic system presented by Lai and Chen in 2016 was synchronized with active control method. Consequently, it is shown that the Lai-Chen chaotic system can be synchronized and used in chaos-based communication and encryption applications.

Keywords: *Lai-Chen chaotic system, chaotic synchronization, active control synchronization.*

1. Introduction

Unpredictable and irregular events have previously been seen as undesirable. Because of this feature, such events are called "chaos" in the sense of "unknownness, confusion" [1]. Chaos has complex behavior, as well as a unique structure. Chaos has a regularity in irregularity. When chaos behavior is not fully known, chaos is described as an undesirable condition in systems. When chaos behavior is well understood, chaos has begun to be utilized [2].

The meteorologist Lorenz, while studying weather forecasts, unwittingly discovered the existence of chaos. The mathematical model of chaos was first published in 1963 by Edward Norton Lorenz [3]. After the discovery of chaos, studies about chaotic systems have increased. Chaotic systems have the following characteristics: very sensitive dependence on initial conditions and parameters, unstable, bounded, unpredictable [4, 5]. Because of the features, chaotic systems are used in a wide variety of sciences such as encryption [6], communication [7-9], random number generators [10], biology [11], economy [12]. The chaotic systems which have different behaviors and different characteristics for the purpose of using in the mentioned science branches are introduced in the literature [13-15].

In order for the newly introduced chaotic systems to be used in chaos based communication and chaos based encryption applications, the chaotic systems in the receiver and transmitter units need to be synchronized. Several methods have been developed in the literature for synchronizing chaotic systems. Pecora-Carroll [16], adaptive control [17], passive control [18], sliding mode control [19], backstepping design [20] and active control [21-23] are some of these methods.

In this study, a synchronization design was carried out to use chaos based communication and encryption applications of a new chaotic system introduced by Lai and Chen in 2016 [24]. Active control method is used for synchronization.

2. Numerical Simulation of Lai-Chen (2016) Chaotic System

The Lai-Chen chaotic system was introduced in the literature in 2016 [24]. The system consists of three non-linear elements and three parameters *a*, *b*, *c*, *k*. The mathematical expression of the system is given in Eq. 1 [24].

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= byz - cx \\ \dot{z} &= k - y^2 - xy \end{aligned} \tag{1}$$

The parameter values of the chaotic system (1) are $a = 7$, $b = 4.5$, $c = 8$ and $k = 13$. The block diagram designed in the Matlab-Simulink program for numerical simulation of the chaotic system (1) is given in Figure 1.

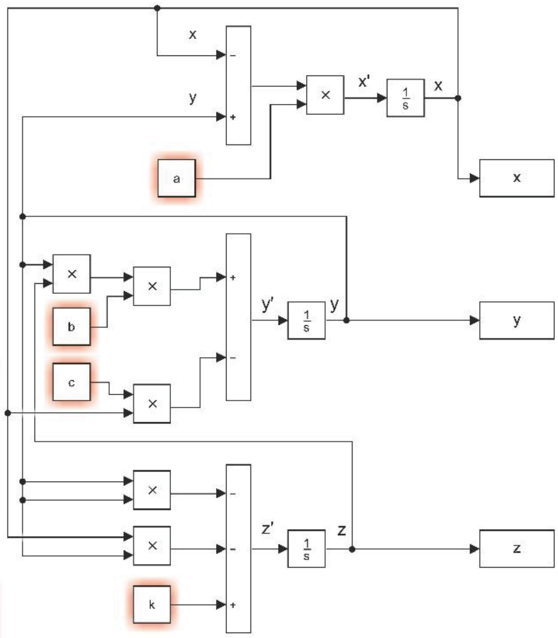
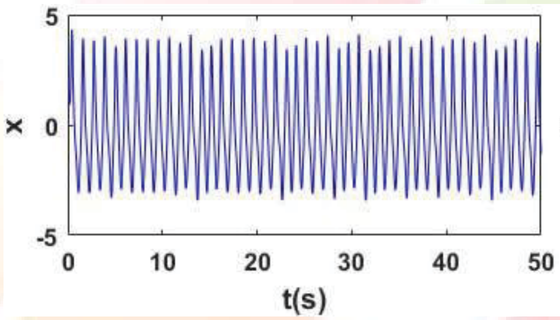
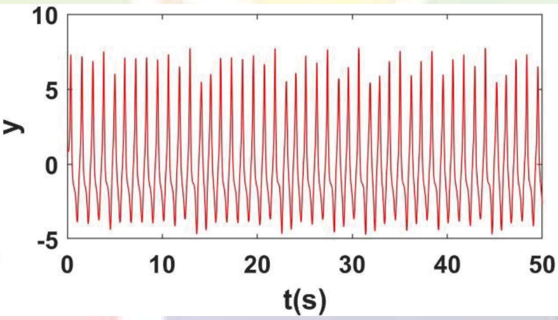


Figure 1: The Matlab-Simulink block diagram for numerical simulation

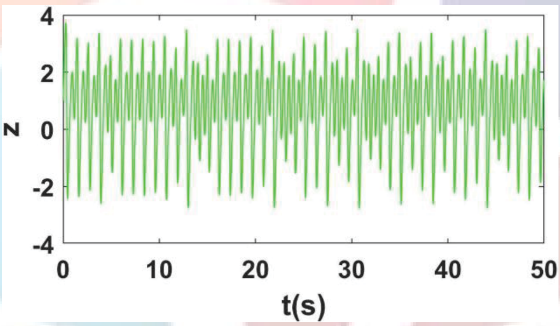
The chaotic system (1) state variables outputs and phase portraits at the initial conditions of X_0=1, Y_0=1, Z_0=1 analyzed in Matlab-Simulink program and given in Figure 2 and Figure 3, respectively.



(a)

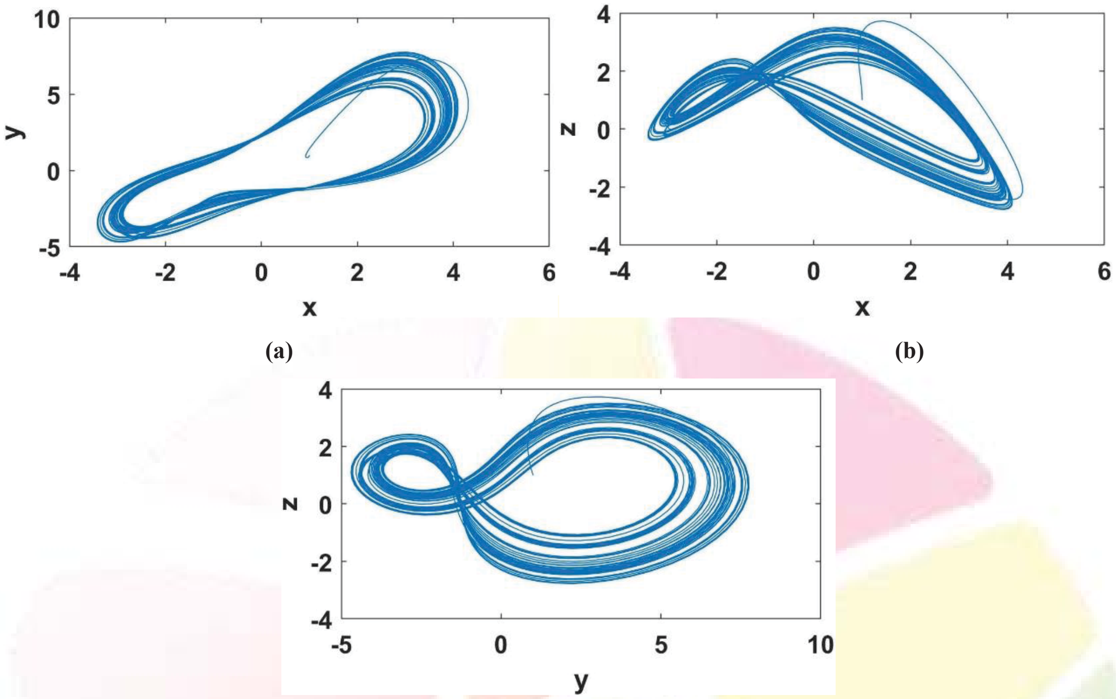


(b)



(c)

Figure 2: State variables outputs of the chaotic system against to time (a) ***x*** **(b)** ***y*** **(c)** ***z***



(a) (b)

(c)

Figure 3: The chaotic system (1) phase portraits (a) *x* versus *y* (b) *x* versus *z* (c) *y* versus *z*

3. Synhcronization of Lai-Chen (2016) Chaotic System with Active Control Method

In active control method for synchronization, one of the two chaotic systems having different initial conditions is called the master and the other is the slave. The mathematical expression of the master system in Eq. 1 and the slave system in Eq. 2 are given. In the slave system (3) $\mu_x(t)$, $\mu_y(t)$ and $\mu_z(t)$ are active control functions [21-23].

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) \\ \dot{y}_1 &= by_1z_1 - cx_1 \\ \dot{z}_1 &= k - y_1^{\,2} - x_1y_1 \end{aligned} \tag{2}$$

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + \mu_x(t) \\ \dot{y}_2 &= by_2z_2 - cx_2 + \mu_y(t) \\ \dot{z}_2 &= k - y_2^{\,2} - x_2y_2 + \mu_z(t) \end{aligned} \tag{3}$$

The error equations between the master system and the slave system are,

$$\begin{aligned} e_x &= x_2 - x_1 \\ e_y &= y_2 - y_1 \\ e_z &= z_2 - z_1 \end{aligned} \tag{4}$$

In the active control design, the equations of error dynamics are obtained by subtracting the master system (2) from the slave system (3). Error Dynamics of the system (1) are given in Eq. 5.

$$\begin{aligned} \dot{e}_x &= \dot{x}_2 - \dot{x}_1 = a(y_2 - x_2) - a(y_1 - x_1) + \mu_x(t) \\ &= -ae_x + ae_y + \mu_x(t) \\ \dot{e}_y &= \dot{y}_2 - \dot{y}_1 = by_2z_2 - cx_2 - by_1z_1 + cx_1 + \mu_y(t) \\ &= -by_1z_1 + by_2z_2 - ce_x + \mu_y(t) \\ \dot{e}_z &= \dot{z}_2 - \dot{z}_1 = -y_2^2 - x_2y_2 + y_1^2 + x_1y_1 + \mu_z(t) \\ &= y_1^2 - y_2^2 + x_1y_1 - x_2y_2 + \mu_z(t) \end{aligned} \tag{5}$$

The control functions are defined as in Eq. 6. The control functions are chosen to eliminate all x_1, x_2, y_1, y_2, z_1, z_2 terms. $u_x(t)$, $u_y(t)$, and $u_z(t)$ are control inputs.

$$\begin{aligned}\mu_x(t) &= u_x(t)\\ \mu_y(t) &= by_1z_1 - by_2z_2 + u_y(t)\\ \mu_z(t) &= -y_1^2 + y_2^2 - x_1y_1 + x_2y_2 + u_z(t)\end{aligned} \tag{6}$$

If the values of the control functions (6) are written in the error dynamic equations (5),

$$\begin{aligned}\dot{e}_x &= -ae_x + ae_y + u_x(t)\\ \dot{e}_y &= -ce_x + u_y(t)\\ \dot{e}_z &= u_z(t)\end{aligned} \tag{7}$$

If the system is stabilized with control inputs $u_x(t)$, $u_y(t)$, $u_z(t)$ while time is infinite, error values e_x, e_y, e_z converge to zero. Thus, the master system (2) and the slave system (3) can be synchronized. For this reason, a constant A matrix is chosen to control the error dynamics in the active control method [21-23].

$$\begin{bmatrix} u_x(t)\\ u_y(t)\\ u_z(t)\end{bmatrix} = A\begin{bmatrix} e_x\\ e_y\\ e_z\end{bmatrix} \tag{8}$$

For the stability of the synchronization, all eigenvalues of the closed-loop system must be negative according to the Routh-Hurwitz criteria [21-23]. The matrix A selected according to this criterion is given in Eq. 9.

$$A = \begin{bmatrix} -a-1 & -a & 0\\ c & -1 & 0\\ 0 & 0 & -1\end{bmatrix} \tag{9}$$

If the control inputs are calculated using Eq. 8 and Eq. 9,

$$\begin{aligned}u_x(t) &= -e_x(a+1) - ae_y\\ u_y(t) &= ce_x - e_y\\ u_z(t) &= -e_z\end{aligned} \tag{10}$$

If Eq. 10 is used in Eq. 6, the control functions are obtained as given in Eq. 11.

$$\begin{aligned}\mu_x(t) &= -e_x(a+1) - ae_y\\ \mu_y(t) &= by_1z_1 - by_2z_2 + ce_x - e_y\\ \mu_z(t) &= -y_1^2 + y_2^2 - x_1y_1 + x_2y_2 - e_z\end{aligned} \tag{11}$$

A schematic diagram of the synchronization of the Lai-Chen chaotic system (1) with the active control method in the Matlab-Simulink® program is given in Figure 4.

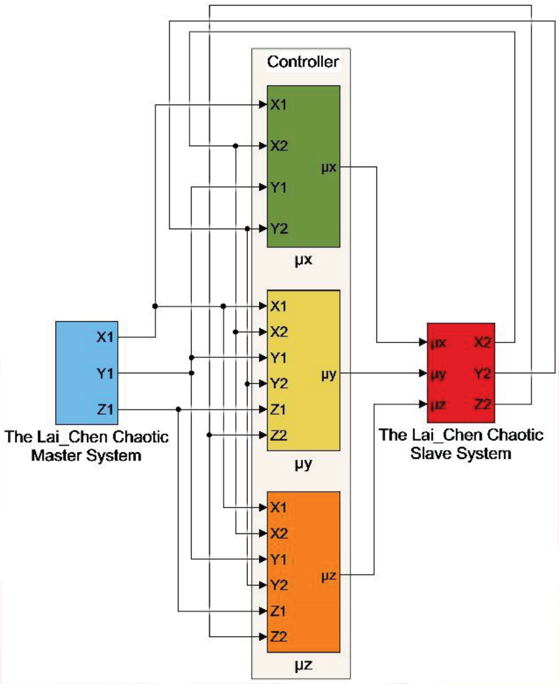
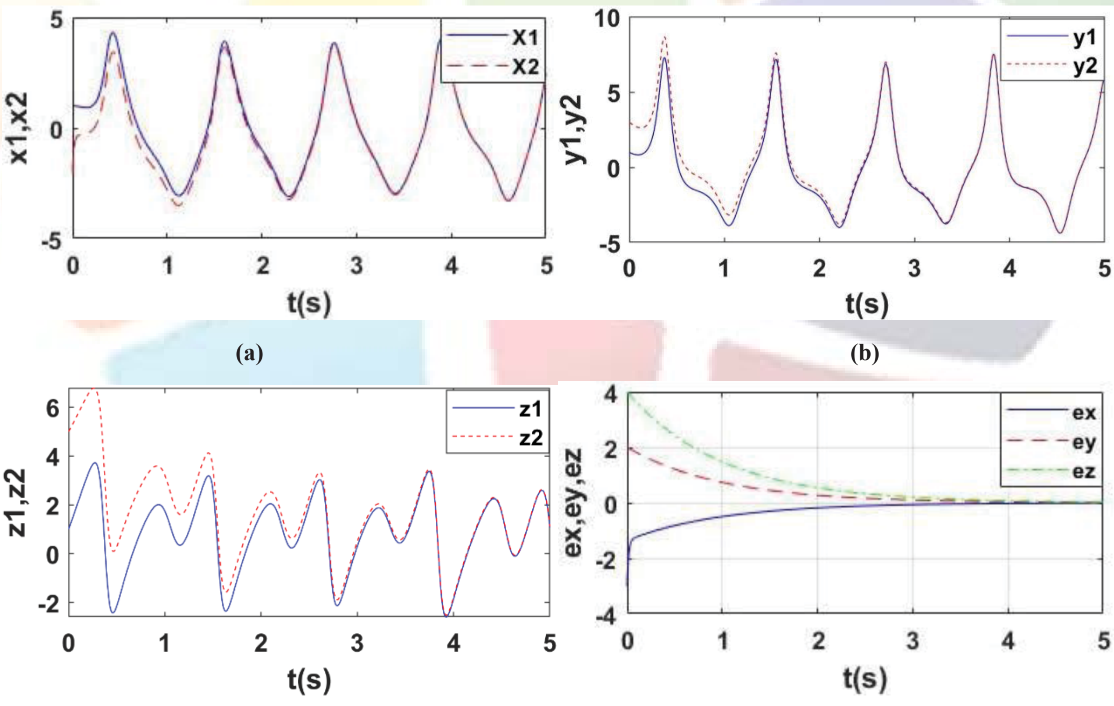


Figure 4: Schematic diagram of the synchronization of the Lai-Chen chaotic system (1)

In the analysis of the synchronization design in the Matlab-Simulink® program (Fig.4), initial conditions of the master system is $X_1 = 1$, $Y_1 = 1$, $Z_1 = 1$ and initial conditions of the slave system is $X_2 = -2$, $Y_2 = 3$, $Z_2 = 5$. Figure 5 shows the outputs of the state variables of the master system x_1, y_1, z_1 and the state variables of the slave system x_2, y_2, z_2 and error dynamics e_x, e_y, e_z with respect to time. The state variables of the master and slave systems are exactly synchronized with each other (Fig 5a, 5b, 5c). It is seen that the error dynamics are zero over time (Fig. 5d). As a result, the Lai-Chen chaotic system was successfully synchronized with the active control method.



(a) (b)

(c) (d)

Figure 5: The outputs of the synhronization design (a) x_1-x_2 (b) (a) y_1-y_2 (c) z_1-z_2 (d) error dynamics e_x, e_y, e_z

4. Results

In this study, a new chaotic system presented by Lai and Chen in 2016 was synchronized with active control method. Numerical simulations of the Lai-Chen (2016) chaotic system were investigated in the Matlab-Simulink program. The mathematical model for the synchronization of the chaotic system with the active control method is calculated. Then, the system designed for synchronization was analyzed in Matlab-Simulink program with different initial conditions. When the analysis results were examined, it was seen that the system was successfully synchronized. As a result, the synchronized chaotic system can be used for chaos-based communication and encryption applications.

5. References

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