ACTA DIDACTICA NAPOCENSIA

**Volume 12, Number 2, 2019 -** DOI: 10.24193/adn.12.2.11

# EXAMINATION OF THE METAPHORICAL PERCEPTIONS OF MATHEMATICS TEACHER CANDIDATES TOWARDS MATHEMATICAL PROVING

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**Abstract:** The purpose of this study is to examine mathematics teacher candidates' perceptions about mathematical proving through metaphors. The participants of the research were 203 mathematics teacher candidates who were having education at the Faculty of Education of the two state universities in the Central Anatolia region, and they were selected on the basis of volunteerism. As a means of data collection, the mathematics teacher candidates were given papers which were written "proving is like..... because....." on. While the metaphors developed by mathematics teacher candidates were being analyzed, the appropriate and inappropriate metaphors were determined. Then, the sources of metaphors developed by mathematics teacher candidates for proving were read in detail, and these metaphors were classified according to their similarities, and categorized. At the end of the content analysis, the metaphors developed by the participants were collected under a total of 12 conceptual categories. When these categories were examined, it was found that 35% of the participants perceived proving as a difficult, complex, boring and unnecessary process, and their perceptions about proving were negative, while the remaining participants (65%) had positive perceptions towards proving, and generally the metaphors obtained were towards explaining the nature of proving.

**Key words:** Mathematics education, mathematical proving, mathematics teacher candidates, metaphorical perception, qualitative research method

## 1. Introduction

Mathematics is a system on its own that consists of structures and connections, and is an abstract concept that includes consecutive abstractions and generalizations (Alakoc, 2003). Also, mathematics is used as a universal language, unlike other branches of science. People are able to understand each other by doing mathematics no matter in which part of the world they live, as mathematical information does not change anywhere in the world. Mathematical language and proving have an important place in mathematics' having this feature (Guler & Dikici, 2012). Harel and Sowder (2007) describe mathematical proving as a mental activity to take suspicion out of a claim. For mathematicians, the aim of mathematical proving is not only to show that a proposition is true or false but also why it is true or false (De Villiers, 2003; Hanna, 2000). In addition, the proving process helps both to understand the meaning of the given theorem, and to show its relation to previous theorems (Lee, 2002). This is because mathematical proving involves processes such as hypothesis, deriving associations, associating concepts, verifying expressions, and generalizing new information (Schabel, 2005). As it is seen, the logical implications, such as reasoning, association and deduction which form the basis of mathematics are at the center of mathematical proving.

Mathematical proving is seen as an important tool for understanding mathematics and doing mathematics as well as verifying knowledge (Almeida, 2000; Ball, Hoyles, Jahnke, & Movshovitz-Hadar, 2002). Because the proving used is also used to show the correctness of the results that students already know and why these results are correct. Proving thus provide a better understanding of mathematical concepts and the development of mathematical thinking (Hanna, 1991; Kitcher, 1984). In addition, students can explore the construction of mathematical knowledge by making experiments

*Received February 2019.*

**Cite as:** Yazlik, D. O. (2019). Examination of the metaphorical perceptions of mathematics teacher candidates towards mathematical proving. *Acta Didactica Napocensia,* 11(2), 145-160, DOI: 10.24193/adn.12.2.11.

in the process of proving, and understand how mathematical results are achieved by way of hypothesis (Stylianides, 2007; Tall, 1998). This allows students to recognize what mathematicians are doing and respect mathematicians, and put themselves in the position of mathematicians to prove a proposition. Proving also helps students develop their character such as patience and persistence. For this reason, National Council of Teachers of Mathematics (NCTM, 2000) emphasizes the need "reasoning and mathematical proving" for the mental development of students starting from the preschool period. When the curriculum in Turkey is looked into, it is seen that a regulation in learning environment is suggested for students to help them to be able to do problem solving effectively, explain their solutions and thoughts considering, to relate mathematics to both within itself and with other fields and to make inferences about logical induction and deduction (Ministry of National Education [MoNe], 2005; Ministry of National Education [MoNe], 2013). Despite the emphasis on mathematical proving in the curriculum, it has been determined in researches that students and mathematics teacher candidates have difficulty in recognizing mathematical proving (Almeida, 2000; Anapa & Samkar, 2010; Arsac, 2007; Aydogdu, Olkun, & Toluk, 2003; Morris, 2002; Nyaumwe & Buzuzi, 2007; Raman, 2003; Recio & Godino, 2001; Stylianides, Stylianides, & Philippou, 2007; Weber, 2004). It has also been determined that the perceptions and experiences of teachers and teacher candidates about mathematical proving directly influence their mathematical proving abilities, and also this influence the students' proving ability gaining processes (Almeida, 2003; Knuth, 2002). For this reason, it is considered important to examine the mathematical proving of mathematics teacher candidates who will train the mathematicians of the future, and also to examine their perceptions and opinions in this regard. When the related studies have been examined, it is seen that the opinions of teacher candidates have been investigated using different techniques. In recent years, the views related to metaphors that are powerful mental mapping tools for understanding and structuring the worlds of individuals have led to an increase in the scientific studies conducted using metaphor analysis (Arslan & Bayrakci, 2006; Ozdemir, 2012). Especially in the field of mathematics education, it is seen that metaphors are used to reveal the concept traces in the minds of the people through analogy with other words not related to the concept (Altintas, 2018; Cansiz-Aktas & Aktas, 2013; Kilic & Yelken-Yalpar, 2013; Latterell & Wilson, 2017; Oflaz, 2011; Yee, 2012). For this reason, it is thought that the research will contribute to filling this gap in the literature. In this framework, the answers for following questions have been seeked:

1. What are the metaphors created by mathematics teacher candidates towards proving?

2. What conceptual categories do these metaphors have in terms of their common features?

## 2. Method

This study, which aims to identify metaphorical perceptions of mathematics teacher candidates towards proving, was carried out by using qualitative research method in the survey model.

### 2. 1. Participants

Participants consist of 203 mathematics teacher candidates trained in the education faculties of the two state universities located in the Central Anatolian region in 2017-2018 academic year. 44 (21.7%) are the first, 54 (26.6%) are the second, 49 (24.1%) are the third and 56 (27.6%) are the fourth grade students. Of these students, 132 (65%) are female and 71 (35%) are male. The participants were determined using the purposeful sampling method. The reason why the participants were composed of mathematics teacher candidates was that they had faced mathematical proving in their field courses, and they would design and apply activities that include proving process in their classes when they start working. The study was conducted at the end of spring term because the participants in the first grade would have completed abstract mathematics and geometry courses. In addition, criteria such as volunteering and easy accessibility were taken into account in the selection of teacher candidates.

## 2. 2. Data Collection

In the data collection process, firstly the participants have been explained what the "metaphor" phenomenon is and how it is used giving samples from the previous studies. Then, in order to determine the mental images in the minds of mathematics teacher candidates, papers written "proving is like.....; because..." on have been given to them. "Like" statement in this sentence is given to reveal the connection between the subject of metaphor and its source, "because" statement is given to make the participants provide justification and reasonable base for the metaphors created by them. Lastly, the participants have been asked to complete the sentence using only one metaphor and explaining its reason, and this have lasted for 15-20 minutes. It is thought that this time limit would be enough as it is aimed the teacher candidates should focus on the first metaphor that comes to their mind first. The answers given by the participants to the above sentence cursively have created the basic data source of the research.

## 2. 3. Data Analysis

The analysis of the data has been analyzed by content analysis method. The analysis process has been carried out in three steps. The papers given to teacher candidates participating in the first step are coded as T1, T2, ..., T203, and the metaphors developed by the participants have been read one by one to determine appropriate and non-appropriate metaphors. For this reason, metaphors are examined in terms of the relation between image of metaphor, the source of metaphor, and subject of metaphor and its source (Saban, 2008). At the end of this step, a total of 52 papers have been excluded due to the reasons described below.

a. Papers that do not contain any metaphors. Here, it is seen that participants haven't used any metaphor and tried to define the proving For example; "Proving is exactly like understanding and inquiring from every direction, because proving is questioning everything." (T54, F)

b. Papers which present no justification for the metaphor. It has been found here that the participants explicitly state the source of the metaphor but do not specify the logical basis of the metaphor. For example, "Proving is like enthusiasm; because ....." (T89, F) as it is seen there is no reason why it is put forward even though the subject of metaphor (proving) and the source of metaphor (enthisuasm) in the image of metaphor is given.

c. Papers containing metaphors that do not contribute to a better understanding of proving. It has been determined that the metaphors in this criterion do not articulate any significant feature attributed to the origin of the metaphor, but to the source of the metaphor. As it is seen in the example "Proving is like building a construction; because you build a structure at the end." (T157, F); although the subject of metaphor (proving) and its source (building) are evident, it is concluded that this metaphor does not contribute to a better explanation, in other words, it is not clear which feature of metaphor is emphasized.

d. Papers containing metaphors belonging to more than one category. According to this criterion, it is seen that the metaphor image in the paper contains several features attributed to the metaphor (to prove), and it is not clear which one of these features is fore grounded. For example, as it is seen in "Proving is like accomplishing; because it is difficult but it gives happiness when accomplished" (T166, M). The image of metaphor as in here, features that belong to two different categories ("proving as a difficult process to accomplish" and "proving as a process that gives happiness when accomplished") are given and because it could not be decided which to accept, it was left out of assessment.

In the second step, the current metaphors from the remaining 151 sheets have been recoded and listed after the 52 papers have been excluded. Later, the sources of these metaphors have been read in detail and grouped under 12 conceptual categories according to their common characteristics. Best names for these categories are given according to the metaphors they contain.

In the third step, the validity and reliability of the study have been achieved. Yildirim and Simsek (2005) stated that in qualitative research, it is an important criterion for validity to report in detail how the data and the results obtained. In this direction, the data collection and analysis process in the

survey is explained in detail. In addition, the sample metaphors of the participants representing the categories for each of the categories obtained in the survey have been included in the findings section. In order to ensure the reliability of the study, the expert opinion have been consulted to confirm that the metaphor images under the 12 conceptual categories obtained at the end of the study represent the conceptual categories indicated. Therefore, a field expert has been given two lists of names of 151 metaphor images and 12 conceptual categories, and it is required to match the categories given in the metaphor information in the list. Later, the matches made by the expert and the researcher have been compared and it is determined that the expert and the researcher matched the four metaphors (solving puzzles, finding the right key, doing philosophy, pomegranate) with different categories. Later, the reliability of the study has been calculated as 97% (147/147 + 4 = 0.97) using the formula by Miles and Huberman (1994). In qualitative studies, it is assumed that the confidence between the expert and the researcher evaluations is sufficient at 90% and above (Miles and Huberman, 1994).

## 3. Findings

At the end of the research, a total of 113 metaphors have been obtained from the 151 mathematics teacher candidates regarding the concept of "proving". It was determined that 20 of these 113 metaphors have been developed by at least 2 and at most 6 teachers, and the remaining 93 metaphors have been developed by only one teacher candidate. Table 1 lists the metaphors developed by teacher candidates and gives the number of teacher candidates representing each metaphor.

**Table 1.** M*etaphors developed in relation to "proving" and the number of teacher candidates representing each metaphor*

| Metaphor name | Frequency (f) | Metaphor name | Frequency (f) |
|---|---|---|---|
| Finding the exit of the labyrinth | 6 | Playing Backgammon | 1 |
| Jigsaw puzzle | 5 | Choosing clothes | 1 |
| Solving puzzles | 5 | Weather forecast | 1 |
| Unbinding the knotted rope | 5 | Flying | 1 |
| Cooking | 4 | Sewing clothes | 1 |
| Climbing stairs | 4 | Eating japanese persimmon | 1 |
| Living | 3 | Understanding politicians | 1 |
| Opening the locked door | 2 | Walking on a thorny path | 1 |
| Knitting | 2 | Making animals speak | 1 |
| Playing a game | 2 | Describing the rainbow to a blind | 1 |
| Death agony | 2 | Walking on the snow | 1 |
| Walking | 2 | Well pumping | 1 |
| Trying to get out of the swamp | 2 | Tell something to an ignorant one | 1 |
| Playing with an brain box | 2 | Starting life from scratch | 1 |
| Building house | 2 | Mountaineering | 1 |
| Matryoshka | 2 | The vertical money position at heads or tail | 1 |
| Space | 2 | Struggling with waves | 1 |
| Swimming | 2 | Understanding women | 1 |
| Coming out on top | 2 | Telling a lie | 1 |
| Water | 2 | Death | 1 |
| Wearing make up | 1 | Finding out why baby is crying | 1 |
| Looking at the back of a mountain | 1 | Realizing dreams | 1 |
| Discovering | 1 | Looking after a child | 1 |
| Candle light | 1 | Our brain | 1 |
| Doing an experiment | 1 | Soup | 1 |
| A person who does not lie | 1 | Lettuce | 1 |
| Making inquiries | 1 | Nerves in the body | 1 |
| Wearing glasses | 1 | Love | 1 |
| Talking to my dad | 1 | Unknown questions | 1 |

| | | | |
|---|---|---|---|
| Being questioned | 1 | Walking alone at night | 1 |
| Doing philosophy | 1 | Base of a building | 1 |
| Travelling by train | 1 | Boat | 1 |
| Dominos | 1 | Tree roots | 1 |
| A book to adapt film | 1 | My mother | 1 |
| Making cookies | 1 | Ingredients in a meal | 1 |
| Navigating a ship | 1 | Soil | 1 |
| Pomegranate | 1 | Parachuting | 1 |
| Reading book | 1 | Newborn baby | 1 |
| Backstroke | 1 | A child's beginning to walk | 1 |
| Paling up with girls | 1 | Drawing a picture | 1 |
| Playing hide and seek | 1 | Towage with toy blocks | 1 |
| Gasoline price hike | 1 | Foreign language | 1 |
| Pen | 1 | Eating chocolate | 1 |
| Nested boxes | 1 | Fasting | 1 |
| Writing a poem | 1 | Learning to ride a bicycle | 1 |
| Counting one's prayer beads | 1 | Achieve success | 1 |
| The digits of Pi after comma | 1 | Sensitive plant | 1 |
| Gasless car | 1 | Woman | 1 |
| Steak tartar a la turca | 1 | Split in a sock | 1 |
| Bus ticket price increase | 1 | Pitched battle | 1 |
| Balloon | 1 | Eating pumpkin meal | 1 |
| Running on a treadmill | 1 | Marriage | 1 |
| Gossiping | 1 | Cleaning | 1 |
| Warming a meal over and over then eating it | 1 | Searching for treasure | 1 |
| Listening to lesson | 1 | Dreaming | 1 |
| Decayed tooth | 1 | Sixth Sense | 1 |
| Finding the right key | 1 | Total | 151 |

## 3. 1. Conceptual Categories

As a result of the analysis, metaphors developed by mathematics teacher candidates towards proving have been classified according to their common characteristics, and collected under 12 categories. Of the 113 metaphors developed, 104 have been under a single category while 9 have been under at least 2 and up to 4 categories. For example, while the metaphor of "solving puzzles" developed by T27 is in the category of "Proving as a process with rules and methods", the metaphors of "solving puzzles" developed by T114 and T139 have been under "proving as a process requiring knowledge" and " proving as a process that is enjoyed when accomplished " categories respectively. The distribution of metaphors developed by teacher candidates according to the categories and the number of teacher candidates representing them are given in Table 2.

**Table 2.** *The categories developed in relation to "proving" concept and the number of teacher candidates representing them*

| Categories | | f | % |
|---|---|---|---|
| 1 | Proving as a process with rules and methods | 41 | 27.2 |
| 2 | Proving as a difficult and complicated process to do | 36 | 23.9 |
| 3 | Proving as a process that shows/proves the truth | 13 | 8.61 |
| 4 | Proving as a process with continuity | 10 | 6.62 |
| 5 | Proving as an enjoyable process when accomplished | 10 | 6.62 |
| 6 | Proving as a boring process | 10 | 6.62 |
| 7 | Proving as the basis of the mathematics | 7 | 4.64 |
| 8 | Proving as a process that requires knowledge | 6 | 3.97 |
| 9 | Proving as an unnecessary process | 6 | 3.97 |
| 10 | Proving as a process that evolves as you do | 5 | 3.31 |
| 11 | Proving as a process that requires devotion | 4 | 2.65 |
| 12 | Proving as a process based on assumptions | 3 | 1.99 |

**3.1.1. Category 1: Proving as a process with rules and methods.** This category represents a total of 41 teacher candidates (27.2%) and 27 metaphors developed by them. Table 3 lists the metaphors gathered under this category and the number of participants who developed these metaphors.

**Table 3.** *The metaphors that constitutes the category "proving as a process with rules and methods"*

| Category 1 | Metaphors | f |
|---|---|---|
| Proving as a process with rules and methods | Jigsaw puzzle | 5 |
| | Climbing stairs | 4 |
| | Cooking | 3 |
| | Solving puzzles | 2 |
| | Knitting | 2 |
| | Walking | 2 |
| | Finding the exit of the labyrinth | 2 |
| | Playing a game | 2 |
| | A book to adapt film | 1 |
| | Making cookies | 1 |
| | Paling up with girls | 1 |
| | Swimming | 1 |
| | Navigating a ship | 1 |
| | Dominos | 1 |
| | Building house | 1 |
| | Living | 1 |
| | Pomegranate | 1 |
| | Reading books | 1 |
| | Playing with a brainbox | 1 |
| | Sewing clothes | 1 |
| | Travelling by train | 1 |
| | Playing hide and seek | 1 |
| | Finding the right key | 1 |
| | Playing Backgammon | 1 |
| | Choosing cloth | 1 |
| | Weather forecast | 1 |
| | Flying | 1 |
| | Total | 41 |

The main features of metaphors representing this category are:

**Feature 1. Proving has rules.** It has been determined that the teacher candidates representing this feature emphasize that the rules of proving has certain stages and that these steps must be done in a sequential manner. Examples of metaphors that reflect this feature are given below.

"*Proving is like touching dominoes, because one progress step by step while proving. There are rules, each step is the continuation of the previous step.* "(T7, M)

"*Proving is like trying to adapt a book to film because the events and people do not change when you film. It does not come out of the book fiction. We can not go out of the rules while proving.* "(T21, F)

**Feature 2: There are methods for proving.** Participants representing this feature have been aware of many proving methods. Below are sample metaphors for this feature.

"*Proving is like playing backgammon, because there are many methods. You can start and play in different ways. Proving is also the same, you can prove it in different ways.* "(T57, M)

"*Proving is like playing hide-and-seek, because there's plenty of place to hide. You can hide in another place every time. There are many different methods that can be used each time when proving a theorem.* "(T14, F)

**3.1.2. Category 2: Proving as a difficult and complicated process to do.** In this category, there are 30 metaphors developed by a total of 36 teacher candidates (23.9%). Table 4 contains metaphors representing this category and the number of participants who have developed these metaphors.

**Table 4.** *Metaphors that constitutes "proving as a difficult and complicated process to do" category*

| Category 2 | Metaphors | f |
|---|---|---|
| Proving as a difficult and complicated process to do | Living | 2 |
| | Death agony | 2 |
| | Trying to get out of the swamp | 2 |
| | Finding the exit of the labyrinth | 2 |
| | Unbinding the knotted rope | 2 |
| | Walking on a thorny path | 1 |
| | Making animals speak | 1 |
| | Describing the rainbow to a blind | 1 |
| | Swimming | 1 |
| | Backstroke | 1 |
| | Soup | 1 |
| | Walking on the snow | 1 |
| | Well pumping | 1 |
| | Telling something to an ignorant one | 1 |
| | Starting life from scratch | 1 |
| | Mountaineering | 1 |
| | The vertical money position at heads or tail | 1 |
| | Struggling with waves | 1 |
| | Understanding woman | 1 |
| | Playing with brain box | 1 |
| | Understanding politicians | 1 |
| | Finding out why baby is crying | 1 |
| | Realizing dreams | 1 |
| | Looking after a child | 1 |
| | Eating Japanese persimmon | 1 |
| | Our brain | 1 |
| | Lettuce | 1 |
| | Nerves in the body | 1 |
| | Love | 1 |
| | Unknown questions | 1 |
| | Total | 36 |

The main features of metaphors representing this category are:

**Feature 1: Proving is difficult.** Some of the metaphors that teacher candidates perceive proving as a difficult process are listed below.

"*Proving is like understanding politicians because you can not understand what they are doing or why they are doing it*" (T13, M)

"*Proving is like telling something to an ignorant one, because it is quite difficult. You can not tell.* "(T62, M)

**Feature 2: Proving is complex.** Some of the metaphors developed by teacher candidates who say proving is a complex are as in the following:

"*Proving is like our brain, because our brain has such a fine detail that it looks complicated. So it is like proving for me*. "(T120, F)

"*Proving is like unbinding a knotted rope, because it's mixed. Fo me proving is too complicated, and it is inextricable.* "(T46, F)

**3.1.3. Category 3: Proving as a process that shows/proves the truth**. This category represents 13 participants (8.61%) and 12 metaphors they produced. Table 5 contains the metaphors that represent this category and the number of participants who have developed these metaphors.

**Table 5.** *The metaphors that constitute "proving as a process that shows/proves the truth" category*

| Category 3 | Metaphors | f |
|---|---|---|
| Proving as a process that shows/proves the truth | Opening a locked door | 2 |
| | Wearing make-up | 1 |
| | Looking at the back mountain | 1 |
| | Exploring | 1 |
| | Candle light | 1 |
| | Doing an experiment | 1 |
| | A person who does not lie | 1 |
| | Making inquiries | 1 |
| | Wearing glasses | 1 |
| | Talking to my dad | 1 |
| | Being questioned | 1 |
| | Doing philosophy | 1 |
| | Total | 13 |

The main features of metaphors gathered under this category are:

**Feature 1: Proving shows the truth.** Teacher candidates representing this feature have been shown to demonstrate the fact that the proving is true. Sample metaphors for this feature are listed below.

"*Proving is like opening a locked door, because you see the truth*." (T3, F)

"*Provingis like looking at the back of a mountain because it shows what's behind it. You find the truth.* "(T6, F)

**Feature 2: Proving is proving the truth.** In this aspect, it has been determined that the teacher candidates prove what they know by proving. Examples of metaphors that represent this feature are given below.

"*Proving is like being questioned, because you need to prove you're right. You can prove a theorem is correct through proving.* "(T55, F)

"*Proving is like doing philosophy, because people have to convince people that their ideas are right, that is, they have to prove it.*" (T97, M)

**3.1.4. Category 4: Proving as a process with continuity.** This category represents 10 teacher candidates (6.62%) and 8 metaphors. Table 6 lists the metaphors that represent this category and the number of participants who have improved these metaphors.

**Table 6.** *The metaphors that constitute "proving as a process with continuity" category*

| Category 4 | Metaphors | f |
|---|---|---|
| Proving as a process with continuity | Space | 2 |
| | Matryoshka | 2 |
| | Pen | 1 |
| | Gasoline price hike | 1 |
| | Nested boxes | 1 |
| | Writing poem | 1 |
| | Counting one's prayer beads | 1 |
| | The digits of Pi after comma | 1 |
| | Total | 10 |

It is seen that teacher candidates who represent this category have the opinion that proving never ends and proving brings another proving. Examples of some metaphors grouped under this category are:

"*Proving is like space because there is no end. IT never ends like space as each proving reveals another proving.* "(T42, M)

"*Proving is like the digits of Pi after comma, because when you find one you look for the next number. As you prove, you prove the other proofs, one ends and the other begins*."(T118, F).

**3.1.5. Category 5: Proving as an enjoyable process when accomplished.** In this category, there are 10 teacher candidates (6.62%) and 9 metaphors they produced. Table 7 contains the metaphors representing this category and the number of participants who developed these metaphors.

**Table 7.** *The metaphors that constitute "proving as an enjoyable process when accomplished" category*

| Category 5 | Metaphors | f |
|---|---|---|
| Proving as an enjoyable process when accomplished | Coming out on top | 2 |
| | Eating chocolate | 1 |
| | Searching for treasure | 1 |
| | Foreign Language | 1 |
| | Learning to ride a bike | 1 |
| | Drawing picture | 1 |
| | Solving puzzle | 1 |
| | Cooking | 1 |
| | Fasting | 1 |
| | Total | 10 |

It has been determined that the teacher candidates who constitute this category are happy and pleased when they complete proving. Some metaphors that represent this category are:

"*Proving is like a foreign language because we can enjoy it when we speak. In the same way when we prove something we enjoy.* "(T25, F)

"*Proving is like cooking, because it gives you happiness when you cook and put it on the table, just as when we write proving is accomplished*." (T111, F)

**3.1.6. Category 6: Proving as a boring process.** This category represents 10 teacher candidates (6.62%) and 9 metaphors they produced. Table 8 lists the metaphors representing this category and the number of participants who have developed these metaphors.

**Table 8.** *Metaphors that constitute the "proving as a boring process" category*

| Category 6 | Metaphors | f |
|---|---|---|
| Proving as a boring process | Finding the exit of the labyrinth | 2 |
| | Unbinding the knotted rope | 1 |
| | Warming a meal over and over then eating it | 1 |
| | Cleaning | 1 |
| | Decayed tooth | 1 |
| | Death | 1 |
| | Eating pumpkin meal | 1 |
| | Marriage | 1 |
| | Listening to lesson | 1 |
| | Total | 10 |

Some of the metaphors that teacher candidates perceive proving as a boring process are as follows.

"*Proving is like listening to a lesson, because it is very boring.*" (T121, M)

"*Proving is like warming a meal over and over and eating, because you get tired of eating the same thing.*" (T31, M)

**3.1.7. Category 7: Proving as the basis of the mathematics.** This category represents 7 teacher candidates (4.64%) and 7 metaphors developed by them. Table 9 gives the metaphors that represent this category and the number of participants who have improved these metaphors.

**Tablo 9.** *Metaphors that constitute the "proving as the basis of the mathematics" category*

| Category 7 | Metaphors | f |
|---|---|---|
| Proving as the basis of the mathematics | Base of a building | 1 |
| | Boat | 1 |
| | Tree roots | 1 |
| | My mother | 1 |
| | Ingredients of the meal | 1 |
| | Soil | 1 |
| | Water | 1 |
| | Total | 7 |

It has been determined that teacher candidates who represent this category think that proving forms the basis of the mathematics and mathematics cannot be without proving. Some examples of metaphors collected under this category are:

"*Proving is like tree roots wherever you look under the ground you see tree roots. It is similar in mathematics that no matter which subject you study there is always proving*. "(T52, F)

"*Proving is like the base of a building, because if there is no base it collapses. Without proving, there will be no mathematics.* "(T12, M)

**3.1.8. Category 8: Proving as a process that requires knowledge.** In this category there are 6 teacher candidates (3.97%) and 5 metaphors they developed. Table 10 lists the metaphors that represent this category and the number of participants who have improved these metaphors.

**Table 10.** *Metaphors that constitute the "proving as a process that requires knowledge" category*

| Category 8 | Metaphors | f |
|---|---|---|
| Proving as a process that requires knowledge | Solving puzzle | 2 |
| | Building a house | 1 |
| | Pitched battle | 1 |
| | Water | 1 |
| | Split in a sock | 1 |
| | Total | 6 |

**Feature 1: Knowledge is required to for proving.** Teacher candidates representing this feature have emphasized that knowledge is needed for proving. Sample metaphors for this feature are listed below.

"*Proving like building a house, because if you do not know how to do it, the building will collapse. Knowledge is needed for proving.* "(T126, M)

"*Proving is like a pitched battle, because navy is necessary to win. Without knowledge there is no proving.* "(T102, M)

**Feature 2: You need to know how to get started to complete proving.** In this aspect, it has been determined that teacher candidates have stated that it is important to know how to start proving in order to prove it. Here are sample metaphors that represent this feature.

"*Proving is like solving puzzles, because next step comes after each step. You must know how to and where to start when proving*. "(T114, F)

"*Proving is like a split in a sock, because if you find the beginning it unravels. You have to find how to start proving that the rest is solved.* "(T149, F)

**3.1.9. Category 9. Proving as an unnecessary process.** This category represents 6 teacher candidates (3.97%) and 6 metaphors developed by them. Table 11 shows the metaphors that represent this category and the number of participants who have developed these metaphors.

**Table 11.** *Metaphors that constitute the "proving as an unnecessary process" category*

| Category9 | Metaphors | f |
|---|---|---|
| Proving as an unnecessary process | Gasless car | 1 |
| | Steak tartar a la turca | 1 |
| | Bus ticket price increase | 1 |
| | Balloon | 1 |
| | Running on a treadmill | 1 |
| | Gossiping | 1 |
| | Total | 6 |

Examples of metaphors developed by teacher candidates who perceive proving as an unnecessary process are given below.

"*Proving is like a gasless car because there is no need for it to work*." (T24, M)

"*Proving is like steak tartar a la turca because you eat it but never feel full, you do proving but it is useless*." (T30, F)

**3.1.10. Category 10: Proving as a process that evolves as you do.** In this category, there are 5 metaphors developed by 5 teacher candidates (3.31%). Table 12 presents the metaphors that represent this category and the number of participants who have improved these metaphors.

**Table 12.** *Metaphors that constitute "proving as a process that evolves as you do" category*

| Category 10 | Metaphors | f |
|---|---|---|
| Proving as a process that evolves as you do | New born baby | 1 |
| | A child's beginning to walk | 1 |
| | Parachuting | 1 |
| | Towage with toy blocks | 1 |
| | Unbinding the knotted rope | 1 |
| | Total | 5 |

It has been determined that teacher candidates representing this category have the opinion that proving skills develop as they continue proving. The sample metaphors in this category are:

"*Proving is like a child's starting to walk because every step after the first step is always better than the previous step. As you do proving, you always prove better than before.* "(T71, F)

"*Proving is like towage with toy blocks, because you do it better every once in a while. It is also true for proving.* "(T82, F)

**3.1.11. Category 11: Proving as a process that requires devotion.** In this category there are 4 teacher candidates (2.65%) and 4 metaphors developed by them. Table 13 presents the metaphors that represent this category and the number of participants who have improved these metaphors.

**Table 13.** *Metaphors that constitute "proving as a process that requires devotion" category*

| Category 11 | Metaphors | f |
|---|---|---|
| Proving as a process that requires devotion | Woman | 1 |
| | Unbinding the knotted rope | 1 |
| | Sensitive Plant | 1 |
| | Achieving success | 1 |
| | Total | 4 |

It has been seen that the teacher candidates in this category have the opinion that proving requires attention, labor and effort. The example metaphors that represent this category are:

"*Proving is like a sensitive plant, because if you do not take care of it, it gets cross, it is to be interested in*." (T28, F)

"*Proving is like unbinding a knotted rope, because not everybody can unbind, it requires effort*" (T69, F)

**3.1.12. Category 12: Proving as a process based on assumptions.** This category represents 3 metaphors developed by 3 teacher candidates (1.99%). Table 14 lists the metaphors that represent this category and the number of participants who have improved these metaphors.

**Table 14.** *Metaphors that constitute "proving as a process based on assumptions" category*

| Category 12 | Metaphors | f |
|---|---|---|
| Proving as a process based on assumptions | Dreaming | 1 |
| | Sixth Sense | 1 |
| | Telling lie | 1 |
| | Total | 3 |

It has been seen that teacher candidates representing this category emphasize that proving is based on assumptions. The sample metaphors in this category are as follows:

"*Proving is like sixth sense, because even if we can not feel it with our five senses, we know its existence. We assume.* "(T91, F)

"*Proving is like dreaming for me, because while dreaming we think the things as real. While proving we always accept.* "(T18, F)

## 4. Result and Discussion

In this study it is aimed to reveal the perceptions of mathematics teacher candidates about proving through metaphors. When the findings obtained at the end of the study have been examined, it has been found that 27.02% of participants have stated that proving has rules and methods, 8.61% stated that proving shows the truth, 6.62% stated that proving had continuity, 4.64% stated that proving is the basis of mathematics and 1.99% stated that proving is based on assumptions. It has been concluded that the metaphors developed by about half of the participants (49.1%) have intended to explain the nature of proving. Similarly, in the study by Zhou and Bao (2009), it has been determined that the majority of mathematics teachers have the opinion that reasonable rules should be followed when proving. Once again, Cansiz-Aktas and Aktas (2013) and Morrow (2004) have concluded in their study that students at mathematics department perceive proving as a systematic process that takes the right actions step by step. In addition to these, in previous studies it has been determined that proving is perceived as a process that confirms mathematical knowledge. and clarifies it (Cansiz-Aktas & Aktas, 2013; Guler & Dikici, 2012; Hanna, 2000; Varghese, 2009). It is also remarkable that very few of the participants (4.64%) indicate that proving is the basis of mathematics. Because, according to many mathematicians and mathematics educators, mathematical proving is an important building stone of mathematics (Hanna, 2000).

Another finding obtained at the end of the research is that the mathematics teacher candidates perceive proving as difficult (23.9%), boring (6.62%) and unnecessary (3.97%). In the light of this finding, it is concluded that 34.5% of participants have negative perception about proving. Similarly, in previous studies it is determined that students and teacher candidates at all levels have difficulty in proving, they do not like it and also find it boring (Almeida, 2000; Anapa & Samkar, 2010; Aydoğdu et al., 2003; Morris, 2002; Nyaumwe & Buzuzi, 2007; Raman, 2003; Recio & Godino, 2001; Stylianides et al., 2007; Weber, 2001; Weber, 2004). However, as a result of the literature search, it is found, some studies that have concluded that proving is a process that is memorized, have no contribution and unnecessary (Doruk & Güler, 2014; Knuth, 2002; Morali, Ugurel, Türnüklü, & Yesildere, 2006). In spite of that, some participants (6.62%) have expressed that they enjoyed when they complete proving. In parallel with this finding, Cansız-Aktas and Aktas (2013) have found in their study that 9.8% of students at mathematics department have enjoyed proving when they succeed it. However, in their study, Nyaumwe and Buzuzi (2007) have determined that 41% of teachers enjoy mathematical proving which is a much higher rate than this study.

Another finding at the end of the study is to determine that participants perceive proving as a process that requires knowledge (3.97%), experience (3.31%) and devotion (2.65%). It can be argued that some participants (9.9%) emphasize that individuals need to be prepared from cognitive and emotional aspects in order to be able to do proving. Similarly, it has been seen in the study conducted by Cansız-Aktas and Aktas (2013) that proving is perceived as a process requiring patience, effort, care and attention. Besides, in the researchers conducted to determine the difficulties students encounter while proving it is seen that students have difficulty in proving because they cannot express the definitions, they do not know what kind of a proving structure they should use, they cannot understand mathematical language and notations and also they do not know how to start proving (Knapp, 2005; Moore, 1994).

## 5. Suggestions

It is thought that the views of teacher candidates and teachers on proving are important. Because these views will influence activity designing which include proving and its practice in the class (Iskenderoglu & Baki, 2011). For this reason, teacher candidates and teachers should be aware of the importance of proving which is seen as the building stone of mathematics. Besides this, it is expected that they should have a high level of proving ability and self-sufficiency. However, it is determined that 34.5% of the teacher candidates perceive proving as difficult, complex, boring and unnecessary process. In order to transform these negative thoughts into positive, it should be ensured that students meet with proving at undergraduate level. For this reason, activities that include proving designed to improve the skills of associating, rationalization and reasoning should be presented to secondary and high school students. In addition, teacher candidates should be encouraged to do proving in teacher training institutions, to discover the aesthetics in proving, and the importance of mathematical proving in mathematics teaching and the ways of teaching mathematical proving should be emphasized. It is also thought that further studies which determine at which point students and teacher candidates have difficulty in mathematical proving process and also at which point they show negative attitudes will contribute the field.

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