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The Effect of Problem Posing Approach to the Gifted Student's Mathematical Abilities

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Abstract

Improvement and development of mathematically gifted students' mathematical abilities has always been one of the main duties of secondary school mathematic teachers in Kazakhstan. Because, The Ministry of Education organizes the Republic Olympic competition at mathematics every year. Secondary school teachers try to develop the mathematical skills of their students. The aim of this study is to give some directions for teachers to move up their students' level by using a problem posing approach which has two dimensions. First special problem posing tasks were prepared for students and second face to face interaction with them. As a result, the usefulness of this approach will be discussed for secondary school teachers in order to use method in their special courses and second how a special curriculum can be prepared for the gifted students. Finally the method of problem posing may be used in the identification process of a gifted student.

Key Words: Problem posing, mathematical, gifted students, learning, capability

Introduction

A primary goal of mathematics teaching and learning is to develop the ability to solve a broad range of complex problems (Stanic, Kilpatrick, 1988). One of the most important emphases in NTCM standards (1989) is to make problem solving or problem posing a central focus of school mathematics. Also recent recommendations for reform in mathematics education suggest the inclusion in instruction of activities in which students generate their own problems in addition to solving pre-formulated problems (NTCM, 2000). Because the problem posing is the main target of mathematics education, our way should be what is really problem posing. In literature there are many definitions of problem posing. Problem posing involves the generation of new problems about a situation or the reformulation of a given problem (English, 1997; Silver and Cal, 1996). In

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our study we don't use problem solving itself "because problem posing is intellectually a more demanding task than solving problem tasks" (Mestre, 2002). By ordinary problem solving, students just use some ways that are learned before. They use the steps to reach the result. But problem solving is not a linear method. Research studies provided evidence that problem posing has a positive influence on the students' ability to solve word problems (Leung and Silver, 1997) and provides the opportunity for teachers to get an insight of students' understanding of mathematical concepts and processes (English ,1997). It was also found that students' experience with problem posing enhances their perception of the subject and produces excitement and motivation (English, 1998: Silver, 1994). Shortly, Problem posing has many effects on students 'thinking, problem solving skills, attitudes and confidence in mathematics and mathematical problem solving and contributes to a broader understanding of mathematical concepts (English, 1998). Because problem posing also involves problem solving we prefer the tasks that are compatible with posing. Stoyanova (2000) identified three categories of problem posing experiences that can increase student's awareness of different situations to generate and solve mathematical problems 1.free situations 2.semi-structured situations and 3.structured problem –posing situations. In free problem posing situations students pose problems without any restriction. An example of the free problem posing situations.

Are the tasks where students are encouraged to write problems for friends to solve or write information by giving it meaning or creating relations between provided information? Semi-structured and structured situations in which students are asked to generate problems from a presented stimulus that includes quantities information. Problem posing is also described by Dunker (1945) as the generation of a new problem or the formulation of a given problem. In the same meaning Silver (1993) described and added posing occurs before, after and during the problem solving.

Problem posing skills could be developed by giving students an ill-formulated or a partially formulated problem and asking them to restart it (Silver, Kilpatrick and Shlesinger, 1990). Although the idea is not new, not much more work done on problem posing. Dillon (1988) and Krutetskii (1976) has the same way of manipulating the given

conditions and the goals of prior posed problems. Hashimoto (1987) asking "What-if" and "What if not" and Brown Walter (1983), assuming new relationship of supply new story components during the looking back stages (Polya, 1945).

Gifted Students

The definition of a mathematically gifted student is really difficult. Because every student is unique. However, diverse definitions are made by many researchers (Bluton, 1983; Miller, 1990: Gagne, 1991) and the definition of Gagne is more suitable for our study as "students who are distinguished by experts to have excellent ability and potential for great achievements". Other educational researches (Kruetskii, 1976; Sriraman, 2003; Lee, 2005) that observed and analyzed the thinking characteristics of mathematically gifted students, their problem solving and reasoning are displayed very differently from those of ordinary students in terms of speed and depth. In our study we mean the gifted student as students with high mathematical abilities. Meanwhile, mathematically gifted students have ability to assume critical roles as creative contributors because of their insightful reasoning. Gifted students are not satisfied with school tasks that are limited scope in many cases Costa (1998). Silver and Stein (1996) reports that if regular instruction is given to the gifted students, they show higher performances at high level thinking and reasoning. The ability of experts who may be gifted children has also different attributes when they are engaged in problem solving. (Hoose, 1987; Johnson, 1983) stated some characteristic properties of gifted students as; they have exceptional memory, they have ability to solve problems in unexpected way, they have success in identifying patterns and relationship, they enjoy posing original problems, they like to study abstractly, they are learning rapidly, they conserve a long concentration span when they are interested, they have a capacity for self directed to any activity, they have a preference for mathematical activity and they enjoy making mathematical puzzles and games. In short, the quality of instructional tasks provides an opportunity for gifted students to develop mathematical abilities through high level thinking and reasoning (Henningsen and Stein, 1997). The aim of the study is similar with the idea stated above. The problem posing activities given to the students has positive effect on the development of abilities of the gifted ones.

Method

Design

The study aims that how would be correlation between problem posing instructions with designed problem posing tasks and Mathematical Problem Solving Ability Test (MPSAT). Mathematical ability tests were used as pre and post tests in the study. Before problem posing instruction, a pre-test was given to all students. And students were divided into two groups as experimental and control groups. Then a mathematical post test was given to both groups after problem posing instruction.

Participants

The participants of this study are 40 eighth grade students in a special school that is namely called "school for Kazakh gifted students". They were given a special mathematics program that was regulated by Ministry of Education. The students have to pass the entrance exam in order to be a student in this school. Participants had passed this special exam last year and finished the prep class. The students are all candidates for Olympics competitions will be held at the end of the year.

Procedure

The study takes 16 hours over 8 weeks with problem posing activities. In the classroom atmosphere, the approach of students to each question is analyzed. The first question was given to every student for 5 minutes and question is written to the board. After each 5 minutes, teacher asked many questions to the class. Because the questions are prepared according to posing techniques. There are several of sub questions for each posing questions. Although we were trying to measure the effect of problem posing instruction on the students with having high mathematical ability, not all questions from the area of mathematics that is the nearly half of the questions from physics and statistics. One assistant person during problem posing stages took notes by just putting thick if the student answers the question. Moreover, the study involves distinguishing high mathematically gifted students by again problem posing methods.

Purpose of the study

1. To measure the effect of problem posing instruction on the development of mathematical abilities of gifted students.

2. To identify the students with the high mathematical potentials by using the problem posing method(s). And compare the selected students' results with the Mathematical Problem Solving Ability Test (MPSAT)

3. To develop a suitable differentiated mathematical curriculum for Olympiads Programs.

4. How the mathematics teachers use problem posing techniques in their special course of Olympiads preparation.

Data collection and analysis

Data is collected by the teacher and assistant person during the problem posing process. A mark is put to the student when he answers the question. At the end of each lesson scores are added and students are evaluated. Before problem posing instruction, the students took a mathematical ability test as a pre-test. And after 8 weeks, the students were again given Mathematical Problem Solving Ability Test (MPSAT). The results of classroom activities and (MPSAT) are compared to investigate whether a statistically significant relationship exists or not. Results of pre and post test are compared by SPSS computer program by independent t-test. In data analysis, the correlation of problem posing abilities and Mathematical Problem Solving Ability Test (MPSAT) is investigated. The paired sample t-test was performed to investigate the effect of problem posing methods to mathematical problem solving ability test (MPSAT).

Findings

In this research the problem posing tasks are prepared from the identification of Stoyanova (2000) who stated three categories of problem posing experiences that can increase the student's awareness of different situations to generate and solve mathematical problems free situations, semi -structured situations and structured problem- posing situations. The answers of students are collected and evaluated and the

average for each student is processed to the program SPSS. At the end of the course a mathematical ability tests average results with the same manner is loaded.

Some Examples of Tasks Used In Problem Posing Activities

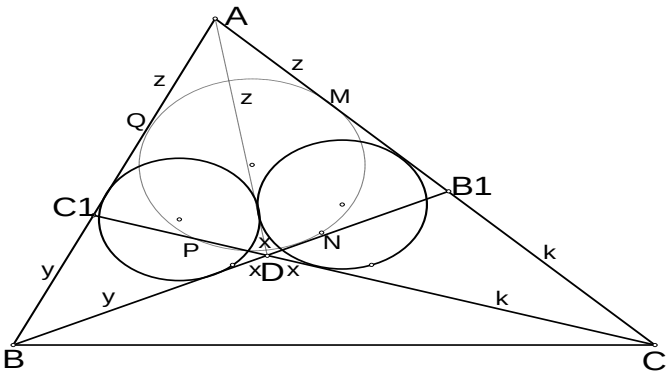
This task is suitable with problem posing oriented courses.

Task 1. B_1 and C_1 are the points on AC and AB sides of the triangle ABC. The lines BB_1 and CC_1 are intersect at D. Prove that AB_1DC_1 is circumscribed if and only if the in circle of ABD and ACD are tangent.

First: The students write the attributes and try to draw figure under the consideration of teacher.

Second: The definitions of mathematical terms will be discussed

Shape 1. Definitions of mathematical terms

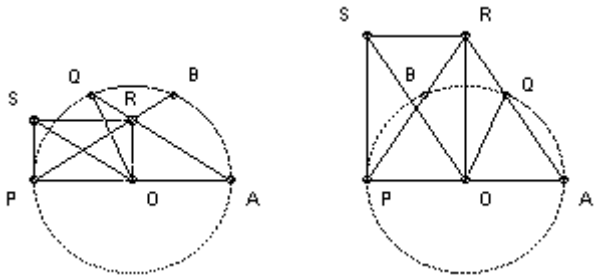


Task 2. QB is a chord of the circle parallel to the diameter PA. The lines PB and QA meet at R. S is taken so that PORS is a parallelogram (where O is the center of the circle).

Show that SP = SQ.

By symmetry, OR is perpendicular to PA. So m<OPR = m<OAR = m<PAQ (same angle) =½m<POQ. But PORS is a rectangle, so m<POS = m<OPR. Hence S lies on the bisector of the angle POQ and the result follows.

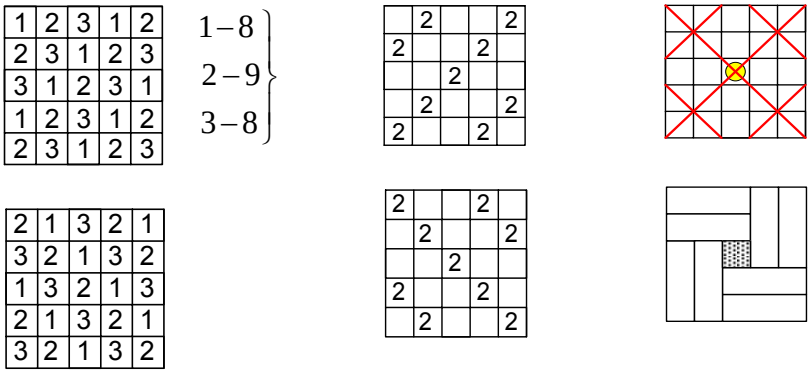
Shape 2. Definitions of mathematical terms



Task 3. This task is difficult for the students but anyway verbally we listen to the student's approach to the task here. A spherical planet has finitely many towns. If there is a town at X, then there is also a town at X', the antipodal point. Some pairs of towns are connected by direct roads. No such roads cross (except at endpoints). If there is a direct road from A to B, then there is also a direct road from A' to B'. It is possible to get from any town to any other town by some sequence of roads. The populations of two towns linked by a direct road differ by at most 100. Show that there must be two antipodal towns whose populations differ by at most 100.

Task 4. After cutting only one square from 5x5 square we can fill out the remaining part using linear triminoes. Determine all the possible squares that are cut off. Since each triminoe covers each number we must remove one of 2's. So we have 9 alternatives but we can eliminate by symmetry.

Shape 3. Definitions of mathematical strategy



Therefore if we combine these two figures we get so the center is the only case.

Table 1. Results of pre and post tests (pre1.control and pre2.experiment)

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	PRE1	5.8250	40	1.83816	.29064
	PRE2	5.9000	40	1.08131	.17097
Pair 2	POST1	5.2812	32	1.57058	.27764
	POST2	6.8750	32	1.31370	.23223

Table one shows that before instruction, there is no significant between the means of the groups. They are nearly the same with an averages 5.8 and 5.9 . After problem posing instruction for group 2 and traditional instruction for group 1, there is a significant difference between means with 5.2 and 6.8. The tasks that is very difficult for both groups and same for students. Here you may say that problem posing instruction is the reason for this increase, especially for non routine and open ended problems.

Table 2. Correlations between pre-pre and post-post tests

		N	Correlation	Sig.
Pair 1	PRE1 and PRE2	40	.442	.004
Pair 2	POST1 and POST2	32	.236	.193

There is no correlation between the pre-tests but there is a correlation between the results of post tests. Table 3 show that t is not significant. That means there are not significant differences between students in control and experimental group according to results of pre test. This is before problem posing instruction. But after problem posing instruction t-value of t is significant and there are differences between the means scores of mathematical ability test for students in experimental group. The problem posing activities are effective in their performances especially for non routine and open ended tasks.

Table 3. Mathematical ability test for students (t-value and significant)

		Paired Differences					t	df	Sig.2-tailed
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	PRE1 and PRE2	-.0750	1.670	.264	.609	.459	.284	39	.778
Pair 2	POST1 and POST2	-1.593	1.793	.317	2.240	-.947	5.027	31	.000

Results and Discussion

The results show that problem posing method used in the identification of gifted student's mathematical abilities and development of these abilities are very useful technique that teachers can use before they start to prepare their students for any olympiad preperation. Our finding is similar with Kuretetskii (1976) who stated that the mathematical high abilities are logical thought and logical reasoning. Because problem posing enriches the verbal ability and provides special ways to explain the solutions, it fosters analysis and thought. Problem posing teaching method is not simply applying and practicing algorithms presented by the teachers, but the students are motivated to challenge new problems and develop their flexible thinking. In order to make successful identification for each individual mathematical abilitiy and develop or foster the identified ability, the teachers also are more familiar with the problem posing stages that were defined by Stoyanova. To develop the mathematical ability for a gifted student is to solve unstandard and non-routine problem problems that are required from the person who solves it to apply mathematical reasoning and algorithms (J.Dunlap, 2001). Our findings are also similar with this idea and problem posing method also increases and fosters mathematical ability because by using this approach the students challenge the existing knowledge. In our study, we used the problems that are more reliable with the types of problem posing that is very important according to Brown and Walter. Because these types of problems develop mathematical thinking and mathematical abilities in students. By this way, the students have to analysis and synthesis the mathematical problems at a high rate. The

research also shows that by problem posing, the interaction between teacher and student is so high and student is so active that is why the teacher can find the gifted student and his or her mathematical ability. Because some part of the research based on the face to face interaction the students' way of thinking can easily be discovered by teachers. Another aim of this research is to adapt curriculum for gifted students at all levels. This can also be achieved by identification of the mathematical abilities of gifted students. Of course, this is very difficult and sometimes a challenge for the existing ones but anyway at least a special curriculum should be adapted for them. As a result, problem posing may be used in the identification process of a gifted student among students or to identify the mathematical abilities for a student. Not only used in the identification process but also it is used to improve and development of mathematical abilities as well.

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