T.C.

NEVŞEHİR HACI BEKTAŞ VELİ ÜNİVERSİTESİ

FEN BİLİMLERİ ENSTİTÜSÜ

# ÜÇ BOYUTLU İKİNCİ MERTEBEDEN FARK DENKLEM SİSTEMİNİN ÇÖZÜMLERİ ÜZERİNE

Tezi Hazırlayan

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Prof. Dr. Yasin YAZLIK danışmanlığında **Arzu YÜKSEL** tarafından hazırlanan "Üç Boyutlu İkinci Mertebeden Fark Denklem Sisteminin Çözümleri Üzerine" başlıklı bu çalışma, jürimiz tarafından Nevşehir Hacı Bektaş Veli Üniversitesi Fen Bilimleri Enstitüsü Matematik Anabilim Dalında **Yüksek Lisans Tezi** olarak kabul edilmiştir.

**JÜRİ**

Başkan :

Üye :

Üye :

Üye :

**ONAY:**

Bu tezin kabulü Fen Bilimleri Enstitü Yönetim Kurulunun………………….tarih ve…………….. sayılı kararı ile onaylanmıştır.

…/…/2022

Prof. Dr. Şahlan ÖZTÜRK

Enstitü Müdürü

## TEZ BİLDİRİM SAYFASI

Tez yazım kurallarına uygun olarak hazırlanan bu çalışmada yer alan bütün bilgilerin bilimsel ve akademik kurallar çerçevesinde elde edilerek sunulduğunu ve bana ait olmayan her türlü ifade ve bilginin kaynağına eksiksiz atıf yapıldığını bildiririm.

Arzu YÜKSEL

# TEŞEKKÜR

Yüksek lisans öğrenimim süresince ve tez çalışmam sırasında tüm bilgilerini benimle paylaşmaktan kaçınmayan, her türlü konuda desteğini benden esirgemeyen ve tezimde büyük emeği olan Sayın Hocam Prof. Dr. Yasin YAZLIK'a,

Maddi ve manevi desteklerini esirgemeyen annem Meryem TEMİZ'e, kardeşim Ayşenur YÜKSEL'e ve arkadaşlarıma,

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**ÜÇ BOYUTLU FARK DENKLEMLERİNİN ÇÖZÜMLERİ**

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## ÖZET

Bu çalışma dört bölümden oluşmaktadır. Birinci bölümde, fark denklemlerinin önemi ve uygulama alanı ile ilgili genel bilgiler verilmiştir. İkinci bölümde, fark denklemleri ve fark denklem sistemleri ile ilgili yapılan çalışmalardan bahsedilmiştir. Ardından fark denklemleri ve fark denklem sistemleri ile ilgili genel tanım ve teoremler verilmiştir. Üçüncü bölümde, $x_{n+1}=\alpha y_n+\frac{ay_n}{y_n-\beta z_{n-1}},\ y_{n+1}=\beta z_n+\frac{bz_n}{z_n-\gamma x_{n-1}},\ z_{n+1}=\gamma x_n+\frac{cx_n}{x_n-\alpha y_{n-1}},\ n\in\mathbb{N}_0,$ fark denklem sistemi tanımlanarak katsayıların durumuna göre 10 alt durumda çözümler elde edilmiştir. Dördüncü bölümde ise sonuç ve öneriler verilmiştir.

***Anahtar kelimeler: Fark Denklemi, Fark denklem sistemi.***

**Tez Danışmanı: Prof. Dr. Yasin YAZLIK**

**Sayfa Sayısı: 74**

**ÜÇ BOYUTLU FARK DENKLEMLERİNİN ÇÖZÜMLERİ**

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## ABSRACT

This study consists of four sections. In the first section, general information about the importance of difference equations and their application area is given. In the second section, studies on difference equations and system of difference equations are mentioned. Then, general definitions and theorems about difference equations and systems of difference equations are given. In the third chapter, the solitions of

$$x_{n+1}=\alpha y_n+\frac{ay_n}{y_n-\beta z_{n-1}},\ y_{n+1}=\beta z_n+\frac{bz_n}{z_n-\gamma x_{n-1}},\ z_{n+1}=\gamma x_n+\frac{cx_n}{x_n-\alpha y_{n-1}},\quad n\in\mathbb{N}_0,$$

are obtained in 10 sub-cases according to the condition of the coefficients. In the fourth section, results and discussions are given.

***Keywords: Difference equation, difference equations system.***

**Thesis Supervisor: Prof. Dr. Yasin YAZLIK**

**Pages: 74**

**İÇİNDEKİLER**

## SİMGELER VE KISALTMALAR LİSTESİ

| | | |
|---|---|---|
| $\mathbb{N}_0$ | : | Doğal sayılar kümesi |
| $\mathbb{N}$ | : | Sayma sayıları kümesi |
| $\mathbb{R}$ | : | Reel sayılar kümesi |
| $\mathbb{C}$ | : | Kompleks Sayılar Kümesi |
| $E$ | : | Kaydırma operatörü |
| $\Delta$ | : | İleri fark operatörü |
| $\Delta^{-1}$ | : | Ters fark operatörü |
| $\bar{x}$ | : | Denge noktası |
| $\sum$ | : | Toplam sembolü |
| $\prod$ | : | Çarpım sembolü |
| $\lfloor . \rfloor$ | : | Tam değer |

# 1. BÖLÜM

## 1.1. GİRİŞ

Fark denklemleri bazen üreteç fonksiyonlarından bazen diferansiyel denklemlerin nümerik yaklaşımlardan doğabildiği gibi bazen de fiziksel bir olayın matematiksel modeli olarak ortaya çıkar [23]. Örneğin bir canlıdaki veya bir doğa olayındaki değişikler genellikle diferansiyel denklemle hesaplanır. Hesaplamalarda, mertebesi birden büyük olan lineer olmayan diferansiyel denklemler büyük önem taşır. Bu tür denklemler biyoloji, fizik, geometri, finans vb. alanlarda sıkılıkla karşımıza çıkar. Ancak diferansiyel denklemlerinin analitik çözümlerini bulmak o kadar kolay değildir. Bu yüzden diferansiyel denklem yerine fark denklemlerini kullanmak daha da önem taşır. Bu nedenle fark denklemleri matematiğin her dalında karşımıza çıkar ve önemi göz ardı edilemez. Fark denklemlerin hesabı diferansiyel denklemin hesabı kadar eskidir, fakat onun kadar gelişmiş değildir. Örneğin; bir tür popülasyon modeli, balık hasadı, kırmızı kan hücrelerinin üretimi, havalandırma hacmi ve kan karbondioksit seviyeleri, basit bir salgın modeli gibi daha birçok konu üzerinde incelemeler fark denklem sistemleriyle yapılmaktadır. Son zamanlarda kararlı bir kontrol sisteminin kararsız hale geldiği fizyolojik bozukluklarla birlikte dinamik hastalıklarla ilişkilendirilmiştir.Bu konudaki ilk makaleler Mackey ve Glass'a aittir.

Sabit katsayılı lineer fark denklemlerin teorik olarak çözülebilir olduğu bilinmektedir ve çözümler için ilk çalışmalar de Moivre, Bernoulli ve Lagrange tarafından yapılmıştır. Fark denklemleriyle ilgili temel bilgiler hemen hemen her kitapta bulunabilir. Denklemler teorik olarak çözülebilir ancak pratik olarak çözülebilir değildir. Genellikle derecesi büyük polinom denklemleri çözmek zordur. Sabit katsayılı homojen lineer fark denklemlerin polinom denklemlerle ilişkili olduğu bilinmektedir ve çözümlerinin zor olduğu da söylenebilir. Öte yandan lineer olmayan fark denklemleri ve sistemleri için çözümleri bulmak daha da zordur. Lineer olmayan fark denklemler ve sistemlerinin neredeyse küçük bir bölümünün dönüşümler yardımıyla çözümleri bilinen denklemler ya da sistemlere indirgenerek çözülebilir olması son zamanlarda oldukça fazla ilgi görmektedir. Dahası bilgisayarların ve programların çıkmasıyla çözülebilir fark denklemleri ve sistemlerinin üzerindeki çalışmalar artmıştır. Bazı bilim insanları bunları kullanarak yeni çözülebilir fark denklemleri ve sistemleri elde etmiştir [34].

Biyoloji, fizik, finans vb. alanlarda karşımıza çıkan bazı örnekler şu şekildedir;

*Hasat Modeli:* Belli bir yasaya göre sürekli çoğalan ve yeterli beslenme kaynaklarına sahip olan bir balık türü hakkında bulunan model $F_{n+1} = rF_n(1-F_n),\ 1<r<3,$ şeklindedir; burada $F_n$ balık sayısını göstermektedir. Bu denklemin $\overline{F}=0$ ve $\overline{F}=\frac{r-1}{r}$ gibi iki denge çözümü vardır. Bunlardan ilki kararsız ikincisi kararlıdır. Her hasat döneminde sabit bir $h \geq 0$ sayısı kadar balık toplandığı kabul edilsin. Böyle bir durumda balık türü hakkındaki denklem $F_{n+1} = rF_n\left(1-F_n\right)-h$ denklemine dönüşür.

Bu denklemin
$$\overline{F}^{(1)}(h) = \frac{1}{2r}\left[(r-1)-\sqrt{(r-1)^2-4rh}\right],$$
$$\overline{F}^{(2)}(h) = \frac{1}{2r}\left[(r-1)+\sqrt{(r-1)^2-4rh}\right]$$
iki denge çözümü vardır. Bu çözümlerin kararlılıkları, $f(x) = rx(1-x)-h$ olmak üzere $A_i = \frac{d_f\left(\overline{F}^{(i)}\right)}{dx} = -2r\overline{F}^{(i)} + r,\ i=1,2,$ yardımıyla incelenebilir. $A_1 = 1+\sqrt{(r-1)^2-4rh}$ olduğundan ilk $\overline{F}^{(1)}$ denge çözümü her $h \geq 0$ ve sabit $r$ için kararsızdır. İkinci denge çözümü için $A_1 = 1-\sqrt{(r-1)^2-4rh}$ olup $|A_2|<1,\ 0 \leq h < h_c$'dir. Burada $h_c = \frac{(r-1)^2}{4r}$'dir. Buna göre $h$ hasatı $h_c$ kritik değerinden daha küçük ise o zaman ikinci denge çözümü kararlı olur. Aksi durumda kararsızdır. Öte yandan, $h = h_c$ için $\overline{F}^{(1)}(h_c) = \overline{F}^{(2)}(h_c) = \frac{r-1}{2r}$ ve $0 \leq h < h_c$ için $0 < \overline{F}^{(1)}(h) < \overline{F}^{(2)}(h)$'dır. Hasat etkisini belirlemek için hasat öncesi ve sonrası denge çözümlerini irdelemek gerekir. Daha önceden bu çözümlerin kararlı olup olmadıkları vurgulandı buna göre $\overline{F}^{(1)}(0)$ ve $\overline{F}^{(2)}(0)$ denge noktalarının hasat sonucu yer değiştirdiği ancak kararlılık özelliklerini korudukları ortaya çıkmış bulunuyor. Ayrıca $0 \leq h < h_c$ aralığında bir hasat toplamak, yani balıkçılığı sürdürebilmek için başlangıçtaki $F_0$ balık sayısı $\frac{r-1}{2r} < F_0 < \frac{r-1}{r}$ şeklinde olmalıdır. Aksi durumda, balık türü yok olmaya mahkumdur. Diğer taraftan, maksimum düzeyde hasattan sakınmak gerekir. Zira böyle bir durumda $F_0$ başlangıç değeri yarı kararlı denge noktasına indirgenir. Bu ise balık nüfusunun tükenme riski altında olduğunu gösterir [4].

*Gerilla-Gerilla Modeli (GGM):* Savaşan iki gerilla kuvvetini göz önüne alalım.

Operasyonel kaybın olmadığı ve kuvvetlerin izole olduğu kabul edilsin. Son kabulün anlamı, her iki kuvvet de dışarıdan ek kuvvet alıyor demektir. Bu durumda GG modeli

$$\begin{cases} \Delta x_n = -g x_n y_n, \\ \Delta y_n = -h x_n y_n, \end{cases}$$

formunu alır. Denklemler taraf tarafa bölünürse, $\frac{\Delta x_n}{\Delta y_n} = \frac{g}{h}$ veya $g y_{n+1} - h x_{n+1} = g y_n - h x_n$ bulunur. Buradan $g y_n - h x_n = L$ 'dir. Burada $L = g y_0 - h x_0 = \text{sabit}$ olur. Eğer $L > 0$ ise $Y$ kazanır, $L = 0$ ise her iki kuvvet yok olur, $L < 0$ ise $X$ kazanır [4].

### 1.2. Amaç ve Kapsam

Bu çalışmanın temel amacı, Stevo Steviç'in 2019 yılında *"On a Class of System of Rational Secon-Order Difference Equation Solvable in Closed Form"* isimli çalışması olan $x_{n+1} = a y_n + \frac{c y_n}{y_n - b x_{n-1}}, y_{n+1} = b x_n + \frac{d x_n}{x_n - a y_{n-1}}, n \in \mathbb{N}_0$, iki boyutlu fark denklem sistemini üç boyutlu $x_{n+1} = \alpha y_n + \frac{a y_n}{y_n - \beta z_{n-1}}, y_{n+1} = \beta z_n + \frac{b z_n}{z_n - \gamma x_{n-1}}, z_{n+1} = \gamma x_n + \frac{c x_n}{x_n - \alpha y_{n-1}}, n \in \mathbb{N}_0$, fark denklem sistemine genişletip, genişletilen üç boyutlu fark denklem sisteminin katsayılarının durumlarına göre çözümlerini elde etmektir.

# 2. BÖLÜM

## 2.1 Kaynak Araştırması

Bu bölümde fark denklemleri ve fark denklem sistemleri ile ilgili literatürde yapılmış olan çalışmalardan bahsedilecektir.

**Kulenoviç ve Nurkanoviç (2005),** *"Global Behavior of a Three-Dimensional Linear Fractional System of Difference Equations"* isimli çalışmalarında

$$x_{n+1}=\frac{a+x_n}{b+y_n},\ y_{n+1}=\frac{c+y_n}{d+z_n},\ z_{n+1}=\frac{e+z_n}{f+x_n},\ n\in\mathbb{N}_0,$$

fark denklem sistemlerinin çözümlerinin global asimptotik davranışlarını incelemişlerdir. Burada $a,b,c,d,e,f$ parametreleri sıfır olmayan pozitif reel sayılar ve $x_0,y_0,z_0$ başlangıç şartları negatif olmayan keyfi sabitlerdir. Ayrıca parametrelerin farklı değerleri için bu sistemin pozitif denge noktalarının bazı koşullar altında global çekimliliğini de incelemişlerdir [27].

**Andruch ve Migda (2006),** *"Further Properties of the Rational Recursive Sequence* $x_{n+1}=\frac{ax_{n-1}}{b+cx_nx_{n-1}}$*"* isimli çalışmalarında

$$x_{n+1}=\frac{ax_{n-1}}{b+cx_nx_{n-1}},\quad n\in\mathbb{N}_0,$$

ikinci mertebeden rasyonel fark denkleminin çözümlerini ve çözümlerin asimptotik davranışını incelemişlerdir. Burada $a$ ve $c$ pozitif parametreler, $b$ negatif parametre ve $x_{-1}$, $x_0$ başlangıç koşulları da negatif olmayan reel sayılardır [1].

**Elsayed (2008),** *"Qualitative Behavior of a Rational Recursive Sequence"* isimli çalışmasında

$$x_{n+1}=ax_n+\frac{bx_nx_{n-1}}{cx_n+dx_{n-1}},n\in\mathbb{N}_0,$$

$x_{-1},x_0$ başlangıç şartları altında ikinci mertebeden fark denklemlerinin çözümlerinin davranışını incelemiştir. Ayrıca bu denklemin birkaç özel durumlarının çözümlerini vermiştir. Burada $a,b,c$ ve $d$ keyfi reel sabitlerdir [13].

**Elsayed (2010),** *"Dynamics of Recursive Sequence of Order Two"* isimli çalışmasında

$$x_{n+1} = ax_n + \frac{bx_n}{cx_n - dx_{n-1}}, n \in \mathbb{N}_0,$$

fark denkleminin çözümlerinin davranışını incelemiştir. Burada $x_{-1}, x_0$ başlangıç şartları keyfi reel sayılardır, $a,b,c,d$ pozitif sabitlerdir ve $cx_0 - dx_{-1} \neq 0$ 'dır. Ayrıca bu denklemin birkaç özel durumlarının çözümlerini vermiştir [12].

**Elabbasy, El-Metwally ve Elsayed (2011)** *"Global Behavior of the Solutions of Some Difference Equations"* isimli çalışmasında $r = \max\{l,k,p,q\}$ negatif olmayan tam sayı olmak üzere $x_{-r}, x_{-r+1}, x_{-r+2}, ..., x_0$ başlangıç şartları altında

$$x_{n+1} = \frac{ax_{n-l}x_{n-k}}{bx_{n-p} - cx_{n-q}}, n \in \mathbb{N}_0,$$

fark denklemini göz önüne almışlardır. Bu denklemin bazı özel durumlarının çözümlerinin davranışını ve çözümlerini elde etmişlerdir. Burada $a,b,c$ pozitif sabitlerdir [8].

**Raouf (2012),** *"Global Behavior of the Rational Riccati Difference Equation of Order Two: the General Case"* isimli çalışmasında

$$x_{n+1} = a + \frac{b}{x_n} + \frac{c}{x_n x_{n-1}}, n \in \mathbb{N}_0,$$

ikinci mertebeden Riccati fark denkleminin çözümlerinin asimptotik davranışı ve kararlılık özellikleri incelemiştir. Burada parametreler *a, b, c* ve $x_{-1}, x_0$ başlangıç koşulları reel sayılardır [33].

**Stevic (2012),** "*On a Third Order System of Difference Equations*" isimli çalışmasında

$$x_{n+1} = \frac{a_1 x_{n-2}}{b_1 y_n z_{n-1} x_{n-2} + c_1}, \; y_{n+1} = \frac{a_2 y_{n-2}}{b_2 z_n x_{n-1} y_{n-2} + c_2}, \; z_{n+1} = \frac{a_3 z_{n-2}}{b_3 x_n y_{n-1} z_{n-2} + c_3}, \; n \in \mathbb{N}_0,$$

üç boyutlu üçüncü mertebeden sabit katsayılı lineer olmayan fark denklem sistemini göz önüne almıştır. Burada katsayıları ve başlangıç koşulları reel sabitlerdir. Bu sistemin katsayılarının durumuna göre çözümlerini dönüşümler kullanarak elde etmiştir [34].

**İbrahim ve Touafek (2013),** *"On a Third Order Rational Difference Equation with Variable Coefficients"* isimli çalışmalarında

$$x_{n+1}=\frac{x_{n-1}x_{n-2}}{x_n\left(a_n+b_nx_{n-1}x_{n-2}\right)},\ n\in\mathbb{N}_0,$$

rasyonel fark denkleminin çözümlerini incelemişlerdir. Burada $a_n,\ b_n$ iki periyotlu periyodik reel dizi ve başlangıç koşulları negatif olmayan reel sayılardır [22].

**Özkan ve Kurbanlı (2013),** *"On a System of Difference Equations"* isimli çalışmalarında

$$x_{n+1}=\frac{y_{n-2}}{\left(-1\pm y_{n-2}x_{n-1}y_n\right)},\ y_{n+1}=\frac{x_{n-2}}{\left(-1\pm x_{n-2}y_{n-1}x_n\right)},\ z_{n+1}=\frac{x_{n-2}+y_{n-2}}{\left(-1\pm x_{n-2}y_{n-1}x_n\right)},\ n\in\mathbb{N}_0$$

rasyonel fark denklem sisteminin periyodik çözümlerini incelemişlerdir. Burada $x_{-2},x_{-1},x_0,y_{-2},y_{-1},y_0,z_{-2},z_{-1},z_0$ başlangıç şartları reel sabitlerdir [32].

**Din (2013),** *"Dynamics of a Discrete Lotka-Volterra Model"* isimli çalışmasında

$$x_{n+1}=\frac{\alpha x_n-\beta x_ny_n}{1+\gamma x_n},\ y_{n+1}=\frac{\delta y_n-\varepsilon x_ny_n}{1+\eta y_n},\ n\in\mathbb{N}_0,$$

ayrık bir Lotka-Volterra modelinin denge noktalarını, denge noktalarının lokal asimptotik kararlılığını ve denge noktalarının global davranışını incelemiştir. Burada $\alpha,\beta,\gamma,\delta,\varepsilon,\eta\in\mathbb{R}^+$ ve $x_0,y_0$ başlangıç şartları pozitif reel sayılardır. Ayrıca tek pozitif denge noktasına yakınsayan bir çözümün yakınsama oranı incelenmiştir. Son olarak teorik sonuçları desteklemek için bazı nümerik örnekler vermiştir [7].

**Elsayed (2014),** *"On the Solutions and Periodic Nature of Some Systems of Difference Equations"* isimli çalışmasında

$$x_{n+1}=\frac{z_{n-1}x_{n-2}}{x_{n-2}\pm y_n},\ y_{n+1}=\frac{x_{n-1}y_{n-2}}{y_{n-2}\pm z_n},\ z_{n+1}=\frac{y_{n-1}z_{n-2}}{z_{n-2}\pm x_n},\ n\in\mathbb{N}_0,$$

rasyonel fark denklem sistemlerinin çözümlerini incelemiştir. Burada başlangıç koşulları sıfırdan farklı reel sayılardır [15].

**Elsayed (2014),** *"Solution for Systems of Difference Equations of Rational Form of Order Two"* isimli çalışmasında

$$x_{n+1}=\frac{x_n y_{n-2}}{y_{n-1}\left(\pm 1\pm x_n y_{n-2}\right)},\ y_{n+1}=\frac{y_n x_{n-2}}{x_{n-1}\left(\pm 1\pm y_n x_{n-2}\right)},\ n\in\mathbb{N}_0,$$

fark denklem sisteminin çözümlerini elde etmiştir. Ayrıca sistemin çözümlerinin periyodikliğini de incelemiştir[14].

**Steviç (2014),** *"On a Cyclic System of Difference Equations"* isimli çalışmasında

$$x_{n+1}^{j}=A+\frac{\left(x_n^{(j+1)}\right)^p}{\left(x_{n-1}^{(j+2)}\right)^q},\ j=\overline{1,k},\ n\in\mathbb{N}_0,$$

k-boyutlu fark denklem sisteminin pozitif çözümlerinin sınırlılığını incelemiştir. Burada $A, p$ ve $q$ pozitif sayılar ve $j+i>k$ ise bazı $j\in\{1,2,...,k\}$ ve $i\in\{1,2\}$ için $x_n^{(j+i)}$, $x_n^{(j+i-k)}$ olarak tanımlamıştır [35].

**Elsayed ve Ahmed (2014),** *"Dynamics of a Three-Dimensional Systems of Rational Difference Equations"* isimli çalışmalarında

$$x_{n+1}=\frac{y_n x_{n-2}}{x_{n-2}\pm z_{n-1}},\ y_{n+1}=\frac{z_n y_{n-2}}{y_{n-2}\pm x_{n-1}},\ z_{n+1}=\frac{x_n z_{n-2}}{z_{n-2}\pm y_{n-1}},\ n\in\mathbb{N}_0,$$

üç boyutlu rasyonel fark denklem sistemlerinin çözümlerini ve çözümlerin periyodikliğini araştırmışlardır. Burada $i\in\{0,1,2\}$ için $x_{-i}, y_{-i}, z_{-i}$ başlangıç koşulları sıfır olmayan reel sayılardır [16].

**Tollu, Yazlik ve Taşkara (2014),** *"On Fourteen Solvable Systems of Difference Equations"* isimli çalışmalarında $x_0$ ve $y_0$ başlangıç koşulları sıfırdan farklı reel sayılar olmak üzere

$$x_{n+1}=\frac{1+p_n}{q_n},\ y_{n+1}=\frac{1+r_n}{s_n},\ n\in\mathbb{N}_0,$$

Riccati fark denklem sistemlerinin 16 durumdan 14 tanesi çözülmüş ve bunlardan 12 tanesi Fibonacci sayıları ile ilişkilendirmişlerdir. Burada $p_n$, $q_n$, $r_n$ *ve* $s_n$, $x_n$ veya $y_n$ dizilerinden biridir [41].

**El-Dessoky ve Elsayed (2015),** *"On a Solution of System of Three Fractional Difference Equations"* isimli çalışmalarında başlangıç koşulları sıfırdan farklı reel sayılar olmak üzere

$$x_{n+1}=\frac{x_{n-2}}{\pm 1+x_{n-2}z_{n-1}y_n},\ y_{n+1}=\frac{y_{n-2}}{\pm 1+y_{n-2}x_{n-1}z_n},\ z_{n+1}=\frac{z_{n-2}}{\pm 1+z_{n-2}y_{n-1}x_n},n\in\mathbb{N}_0,$$

fark denklem sistemlerinin çözümlerini incelemişlerdir [11].

**Steviç, Iricanin ve Smarda (2015),** *"On a Close to Symmetric System of Difference Equations of Second Order"* isimli çalışmalarında

$$x_n=\frac{y_{n-1}y_{n-2}}{x_{n-1}\left(a_n+b_ny_{n-1}y_{n-2}\right)},y_n=\frac{x_{n-1}x_{n-2}}{y_{n-1}\left(\alpha_n+\beta_nx_{n-1}x_{n-2}\right)},\ n\in\mathbb{N}_0,$$

rasyonel fark denklem sisteminin iyi tanımlanmış çözümlerini elde etmişlerdir. Burada $(a_n),(b_n),(\alpha_n)$ ve $(\beta_n)$ dizi ve $i\in\{1,2\}$ için $x_{-i},y_{-i}$ başlangıç koşulları reel sabitlerdir. Elde edilen çözümleri kullanarak çözümlerin asimptotik davranışlarını da incelemişlerdir [36].

**Steviç, Diblik, İricanin ve Smarda (2015),** *"On the System of Difference Equations $x_n=\frac{x_{n-1}y_{n-2}}{ay_{n-2}+by_{n-1}},\ y_n=\frac{y_{n-1}x_{n-2}}{cx_{n-2}+dx_{n-1}}$"* isimli çalışmalarında

$$x_n=\frac{x_{n-1}y_{n-2}}{ay_{n-2}+by_{n-1}},\ y_n=\frac{y_{n-1}x_{n-2}}{cx_{n-2}+dx_{n-1}},\ n\in\mathbb{N}_0,$$

fark denklem sisteminin kapalı formda çözülebilir olduğunu göstermişler ve elde ettikleri formülleri kullanarak çözümlerin asimptotik davranışlarını incelemişlerdir. Burada $a,b,c,d$ parametreler, $i\in\{1,2\}$ için $x_{-i},y_{-i}$ başlangıç şartları reel sayılardır [38].

**Steviç, Alghamdi, Alotaibi ve Elsayed (2015),** *"Solvable Product-Type System of Difference Equations of Second Order"* isimli çalışmalarında

$$z_{n+1}=\frac{w_n^a}{z_{n-1}^b},\ w_{n+1}=\frac{z_n^c}{w_{n-1}^d},\ n\in\mathbb{N}_0,$$

ikinci mertebeden çarpılabilir tipteki iki boyutlu fark denklem sisteminin kapalı formda çözülebildiğini göstermişlerdir. Burada $a,b,c,d\in\mathbb{Z}$ ve $i\in\{0,1\}$ için $z_{-i},w_{-i}$ başlangıç şartları kompleks sayılardır [37].

**Yazlık, Taşkara ve Tollu (2016),** *"On the Solutions of a Three Dimensional System of Difference Equation"* isimli çalışmalarında

$$x_{n+1}=\frac{x_n y_{n-1}}{a_0 x_n+b_0 y_{n-2}},\ y_{n+1}=\frac{y_n z_{n-1}}{a_1 y_n+b_1 z_{n-2}},\ z_{n+1}=\frac{z_n x_{n-1}}{a_2 z_n+b_2 x_{n-2}},\ n\in\mathbb{N}_0,$$

üç boyutlu fark denklem sisteminin çözümlerini dönüşümler kullanarak elde etmişlerdir. Burada $i\in\{0,1,2\}$ için $a_i,b_i$ parametreler ve $x_{-i},y_{-i},z_{-i}$ başlangıç şartları reel sayılardır. Ayrıca elde edilen çözümlerin açık formlarından sistemin iyi tanımlı çözümlerinin asimptotik davranışlarını da incelemişlerdir [44].

**Dekkar, Touafek ve Yazlık (2017),** *"Global Stability of a Third-Order Nonlinear System of Difference Equations with Period-Two Coefficients"* isimli çalışmalarında

$$x_{n+1}=\frac{p_n+y_n}{p_n+y_{n-2}},\ y_{n+1}=\frac{q_n+x_n}{q_n+x_{n-2}},n\in\mathbb{N}_0,$$

üçüncü mertebeden periyodik katsayılı rasyonel fark denklem sisteminin global davranışını incelemişlerdir. Fark denklem sisteminin periyodikliğini, sınırlılığını, yakınsaklık oranını, asimptotik kararlılığını, global çekimliliğini ve global asimptotik kararlılığını incelemiştir. Burada $\{p_n\}$ ve $\{q_n\}$ pozitif sayıların iki periyotlu dizileridir. Ayrıca $i\in\{0,1,2\}$ için $x_{-i},\ y_{-i}$ başlangıç şartları negatif olmayan reel sayılardır [6].

**Elsayed, Alotaibi ve Almaylabi (2017)** *"On a Solutions of Fourth Order Rational Systems of Difference Equations"* isimli çalışmalarında

$$x_{n+1}=\frac{y_n x_{n-2}}{y_n+y_{n-3}},\ y_{n+1}=\frac{x_n y_{n-2}}{\pm x_n\pm x_{n-3}},\ n\in\mathbb{N}_0,$$

dördüncü mertebeden fark denklem sisteminin çözümlerini elde etmişlerdir. Burada $i\in\{0,1,2,3\}$ için $x_{-i},y_{-i}$ başlangıç şartları sıfırdan farklı keyfi reel sayılardır [17].

**Halim (2018),** *"On the Solutions of a Second-Order Difference Equation in Terms of Generalized Padovan Sequences"* isimli çalışmasında $x_{-1}$ ve $x_0$ başlangıç şartları altında

$$x_{n+1}=\frac{\alpha x_{n-1}+\beta}{\gamma x_n x_{n-1}},\ n\in\mathbb{N}_0,$$

rasyonel fark denkleminin çözümü, çözümlerin periyodikliği ve asimptotik davranışı üzerine çalışmıştır. Burada $\alpha, \beta, \gamma$ pozitif reel sayılardır [19].

**Okumuş ve Soykan (2018),** *"Dynamical Behavior of a System of Three-Dimensional Non linear Difference Equations"* isimli çalışmalarında $A \in (0, \infty)$ ve $i \in \{-1, 0\}$ için $x_{-i}, y_{-i}, z_{-i}$ başlangıç koşulları pozitif reel sayılar olmak üzere

$$x_{n+1} = A + \frac{x_{n-1}}{z_n},\ y_{n+1} = A + \frac{y_{n-1}}{z_n},\ z_{n+1} = A + \frac{z_{n-1}}{y_n},\ n \in \mathbb{N}_0,$$

fark denklem sisteminin pozitif denge noktasının global asimptotik kararlılığı, pozitif çözümlerin periyodikliği, direnci ve sınırları çalışılmıştır [31].

**Abo-Zeid ve Kamal (2019),** *"Global Behavior of Two Rational Third Order Difference Equations"* isimli çalışmalarında $x_{-2}, x_{-1}, x_0$ başlangıç şartları altında

$$x_{n+1} = \frac{x_n x_{n-2}}{x_{n-1} - x_{n-2}}, n \in \mathbb{N}_0, \text{ ve } x_{n+1} = \frac{x_n x_{n-2}}{-x_{n-1} + x_{n-2}}, n \in \mathbb{N}_0,$$

iki fark denkleminin de iyi tanımlı tüm çözümlerini ve çözümlerinin asimptotik davranışlarını incelemişlerdir [2].

**Steviç (2019),** *" On a Class of Systems of Rational Second-Order Difference Equations Solvable in Closed Form"* isimli çalışmasında

$$x_{n+1} = ay_n + \frac{cy_n}{y_n - bx_{n-1}}, y_{n+1} = bx_n + \frac{dx_n}{x_n - ay_{n-1}},\ n \in \mathbb{N}_0,$$

lineer olmayan fark denklem sistemini bilinen çözülebilir fark denklemine dönüştürerek kapalı biçimde çözülebildiğini göstermiştir. Burada $a, b, c, d$ parametreler ve $x_{-1}, x_0, y_{-1}, y_0$ başlangıç koşulları reel sayılardır. Ayrıca $(1)\, c = 0; (2)\, d = 0; (3)\, a = 0; (4)\, b = 0$ ve $(5)\, abcd \neq 0$ beş durumu incelemiştir [45].

**Steviç (2019),** *"Solvability of a One-Parameter Class of Nonlinear Second-Order Difference Equations by Invariants"* isimli çalışmasında

$$x_{n+1} = x_n \frac{x_n - x_{n-1} + d}{x_n - x_{n-1}},\ n \in \mathbb{N}_0,$$

tek parametreli lineer olmayan ikinci mertebeden fark denkleminin kapalı formda çözülebilirliğini invaryantları kullanarak göstermiştir. Burada $d, x_{-1}, x_0 \in \mathbb{C}$’dır. Ayrıca, herhangi bir mertebeden fark denklemleri için yöntemin nasıl uygulanabileceğine dair genel bir ipucu da vermiştir [39].

**Steviç ve Tollu (2019)** *“Solvability of Eight Classes of Nonlinear Systems of Difference Equations”* isimli çalışmalarında

$$x_n = \frac{a + p_{n-1}q_{n-2}}{p_{n-1} + q_{n-2}},\ y_n = \frac{a + r_{n-1}s_{n-2}}{r_{n-1} + s_{n-2}},\ n \in \mathbb{N}_0,$$

ikinci mertebeden iki boyutlu fark denklem sistemlerinin sekiz alt durumu için kotanjant dönüşümünü kullanarak sisteminin çözülebilirliğini ve yarı döngülerini incelemişlerdir. Burada $a \in [0, +\infty)$, $p_n, q_n, r_n, s_n$ dizileri $x_n$ ve $y_n$ dizilerinden herhangi biridir, $j \in \{1, 2\}$ için $x_{-j}, y_{-j}$ başlangıç koşulları pozitif reel sayılardır [40].

**Elsayed ve Alzahrani (2019),** *“Periodicity and Solutions of Some Rational Difference Equations Systems”* isimli çalışmalarında

$$x_{n+1} = \frac{y_{n-1}x_{n-2}}{x_{n-2} \pm z_n},\ y_{n+1} = \frac{z_{n-1}y_{n-2}}{y_{n-2} \pm x_n},\ z_{n+1} = \frac{x_{n-1}z_{n-2}}{z_{n-2} \pm y_n},\ n \in \mathbb{N}_0,$$

üç boyutlu üçüncü mertebeden lineer olmayan rasyonel fark denklem sistemlerinin çözümlerini bulmuşlardır. Ayrıca çözümlerin periyodikliğini incelemişler ve sonuçları destekleyen nümerik örnekler vermişlerdir [18].

**Tollu ve Yalçınkaya (2019),** *“Global Behavior of a Three-Dimensional System of Difference Equations of Order Three”* isimli çalışmalarında

$$u_{n+1} = \frac{\alpha_1 u_{n-1}}{\beta_1 + \gamma_1 v_{n-2}^p},\ v_{n+1} = \frac{\alpha_2 v_{n-1}}{\beta_2 + \gamma_2 w_{n-2}^p},\ w_{n+1} = \frac{\alpha_3 w_{n-1}}{\beta_3 + \gamma_3 u_{n-2}^p},\ n \in \mathbb{N}_0,$$

fark denklem sisteminin pozitif çözümlerinin global davranışlarını incelemişlerdir. Burada $u_{-i}, v_{-i}, w_{-i},\ i = 0,1,2$ başlangıç şartları pozitif reel sayılar ve $j \in \{1,2,3\}$ için $\alpha_j, \beta_j, \gamma_j$ parametreler ve $p, q, r$ pozitif reel sayılardır [42].

**Kara, Touafek ve Yazlık (2020) ,** *“Well-Defined Solutions of a Three-Dimensional System of Difference Equations”* isimli çalışmalarında

$$x_{n+1}=\frac{ax_nz_{n-1}}{z_n-\beta}+\gamma,\ y_{n+1}=\frac{by_nx_{n-1}}{x_n-\gamma}+\alpha,\ z_{n+1}=\frac{cz_ny_{n-1}}{y_n-\alpha}+\beta,\ n\in\mathbb{N}_0,$$

üç boyutlu fark denklem sisteminin çözümlerini elde etmişlerdir. Ayrıca çözümlerin davranışlarını incelemişlerdir. Burada başlangıç koşulları ve $a,b,c,\alpha,\beta,\gamma$ sıfırdan farklı reel sayılardır [29].

**Kara, Yazlık ve Tollu (2020),** *"Solvability of a System of Higer order Nonlinear Difference Equations"* isimli çalışmalarında

$$x_n=ay_{n-k}+\frac{dy_{n-k}x_{n-(k+l)}}{bx_{n-(k+l)}+cy_{n-l}},\ y_n=\alpha x_{n-k}+\frac{\delta x_{n-k}y_{n-(k+l)}}{\beta y_{n-(k+l)}+\gamma x_{n-l}},\ n\in\mathbb{N}_0,$$

fark denklem sisteminin kapalı formda çözülebileceğini göstermişlerdir. Burada $k$ ve $l$ pozitif tam sayılar, $a,b,c,d,\alpha,\beta,\gamma,\delta$ parametreleri reel sayılar, $j=\overline{1,k+l}$ için $x_{-j},\ y_{-j}$ başlangıç şartları, reel sayılardır. Ayrıca $l=1$ için çözümlerin asimptotik davranışlarını incelemişler ve elde edilen çözümleri kullanarak iyi tanımlı olmayan çözümlerin kümesini elde etmişlerdir [30].

**Khelifa, Halim, Bouchair ve Berkal (2020),** *"On a System of Three Difference Equations of Higer Order Solved in Terms of Lucas and Fibonacci Numbers"* isimli çalışmalarında

$$x_{n+1}=\frac{1+2y_{n-k}}{3+y_{n-k}},\ y_{n+1}=\frac{1+2z_{n-k}}{3+z_{n-k}},\ z_{n+1}=\frac{1+2x_{n-k}}{3+x_{n-k}},n\in\mathbb{N}_0,$$

yüksek mertebeden rasyonel fark denklem sisteminin genel çözümünün temsili ile ilgili teorik bilgiler vermişlerdir. Burada $n,\,k\in\mathbb{N}_0$, $x_{-k},x_{-k+1},...,x_0,y_{-k},y_{-k+1},...,y_0,z_{-k},z_{-k+1},...,z_0$ başlangıç şartları -3' e eşit olmayan keyfi reel sayılardır. Ayrıca sistemin kapalı formda çözülebilir ve çözümlerin iyi bilinen Lucas ve Fibonacci sayıları ile ilişkili olduğunu göstermişlerdir [26].

**Halim, Berkal ve Khelıfa (2020),** *"On a Three-Dimensional Solvable System of Difference Equations"* isimli çalışmalarında

$$x_{n+1}=\frac{z_{n-1}}{a+by_nz_{n-1}},\ y_{n+1}=\frac{x_{n-1}}{a+bz_nx_{n-1}},\ z_{n+1}=\frac{y_{n-1}}{a+bx_ny_{n-1}},\ n\in\mathbb{N}_0,$$

fark denklem sisteminin çözümünü elde etmişlerdir. Burada $a,b$ parametreler ve $x_{-1},x_0,y_{-1},y_0,z_{-1},z_0$ başlangıç şartları sıfırdan farklı reel sayılardır [20].

**Kara ve Yazlık (2020),** *"On a Solvable Three-Dimensional System of Difference Equations"* isimli çalışmalarında

$$x_n = \frac{z_{n-2}x_{n-3}}{ax_{n-3}+by_{n-1}},\ y_n = \frac{x_{n-2}y_{n-3}}{cy_{n-3}+dz_{n-1}},\ z_n = \frac{y_{n-2}z_{n-3}}{ez_{n-3}+fx_{n-1}},\ n\in\mathbb{N}_0,$$

üç boyutlu fark denklem sisteminin çözülebilirliğini göstermişlerdir. Burada $a,b,c,d,e,f$ parametreler ve $i\in\{1,2,3\}$ için $x_{-i},y_{-i},z_{-i}$ başlangıç şartları reel sayılardır. Ayrıca çözümlerin asimptotik davranışları ile elde edilen formüller kullanılarak sistemi tanımsız yapan başlangıç değerlerinin kümesini bulmuşlardır [23].

**Akrour, Kara, Touafek ve Yazlık (2021),** *"Solutions Formulas for Some General Systems of Difference Equations"* isimli çalışmalarında

$$x_{n+1} = f^{-1}(ag(y_n)+bf(x_{n-1})+cg(y_{n-2})+df(x_{n-3}),\ n\in\mathbb{N}_0,$$
$$y_{n+1} = g^{-1}(af(x_n)+bg(y_{n-1})+cf(x_{n-2})+dg(y_{n-3}),\ n\in\mathbb{N}_0,$$

ve

$$x_{n+1} = f^{-1}\left(a+\frac{b}{g(y_n)}+\frac{c}{g(y_n)f(x_{n-1})}+\frac{d}{g(y_n)f(x_{n-1})g(y_{n-2})}\right),\ n\in\mathbb{N}_0,$$
$$y_{n+1} = g^{-1}\left(a+\frac{b}{f(x_n)}+\frac{c}{f(x_n)g(y_{n-1})}+\frac{d}{f(x_n)g(y_{n-1})f(x_{n-2})}\right),\ n\in\mathbb{N}_0,$$

iki genel fark denklem sisteminin çözümlerinin açık formüllerini vermişlerdir. Burada $f,g:D\to\mathbb{R}, D\subseteq\mathbb{R}$, üzerinde birebir sürekli fonksiyonlardır. Ayrıca $i\in\{0,1,2,3\}$ için $x_{-i},\ y_i$ başlangıç şartları $D$ kümesi üzerinde tanımlı keyfi reel sayılar ve $a,b,c$ ve $d$ parametreleri keyfi reel sayılardır [3].

**Kara ve Yazlık (2021),** *"On Eight Solvable Systems of Difference Equations in Terms of Generalized Padovan Sequences"* isimli çalışmalarında

$$x_{n+1} = f^{-1}\left(af(p_{n-1})+bf(q_{n-2})\right),\ y_{n+1} = f^{-1}\left(af(r_{n-1})+bf(s_{n-2})\right),\ n\in\mathbb{N}_0,$$

fark denklem sistemlerinin çözümlerinin genelleştirilmiş Padovan sayıları ile ilişkili olduğunu göstermişlerdir. Burada $p_n$, $q_n$, $r_n$ ve $s_n$ dizileri $x_n$ ve $y_n$ dizilerinden herhangi biridir, $f:D_f \to \mathbb{R}$, $D_f \subseteq \mathbb{R}$, üzerinde birebir sürekli fonksiyon, $j \in \{0,1,2\}$ için $x_{-j}$, $y_{-j}$ başlangıç şartları $D_f$ üzerinde tanımlı keyfi reel sayılar, $a,b$ keyfi kompleks sayılardır [24].

**Touafek, Tollu ve Akrour (2021),** *"On a General Homogeneous Three-Dimensional System of Difference Equations"* isimli çalışmasında

$$x_{n+1} = f(y_n, y_{n-1}),\ y_{n+1} = g(z_n, z_{n-1}),\ z_{n+1} = h(x_n, x_{n-1}),\ n \in \mathbb{N}_0,$$

üç boyutlu fark denklem sistemlerinin çözümlerinin davranışlarını incelemişlerdir. Burada $x_{-1}, x_0, y_{-1}, y_0, z_{-1}, z_0$ başlangıç şartları pozitif reel sayılar, $f,g,h:(0,+\infty)^2 \to (0,+\infty)$ fonksiyonları sürekli ve sıfırıncı dereceden homojen fonksiyonlardır. Ayrıca bazı yakınsaklık teoremlerini ispatlayarak, tek denge noktasının global asimptotik kararlılığı için uygun koşullar oluşturmuşlar ve sistemin iki asal periyotlu çözümü için gerek ve yeterli koşulları vermişlerdir. Son olarak salınımlı çözümler için de yeni bir sonuç elde etmişlerdir [43].

**Hamioud, Touafek, Dekkar ve Yazlık (2022)** *"On a three Dimensional Nonautonomous System of Difference Equations"* isimli çalışmalarında

$$x_{n+1} = \frac{p_n + y_n}{p_n + y_{n-3}},\ y_{n+1} = \frac{q_n + z_n}{q_n + z_{n-3}},\ z_{n+1} = \frac{r_n + x_n}{r_n + x_{n-3}},\ n \in \mathbb{N}_0,$$

dördüncü mertebeden rasyonel otonom olmayan üç boyutlu fark denklem sisteminin çözümlerinin davranışlarını incelemişlerdir. Burada $\{p_n\},\{q_n\},\{r_n\}$ pozitif sayıların 3 periyotlu dizileri ve $i \in \{-3,-2,-1,0\}$ için $x_i, y_i, z_i$ başlangıç koşulları negatif olmayan reel sayılardır [21].

## 2.2 Ön Bilgiler

Bu bölümde temel bilgiler genel olarak [4], [5], [9] ve [28] kaynaklarından alınmıştır.

**Tanım 2.2.1. [4, 9]** $x:\mathbb{N} \to \mathbb{R}$ tanımlı bir fonksiyon olsun. E öteleme operatörü

$$Ex(n) = x(n+1)$$

şeklinde tanımlanır.

**Tanım 2.2.2. [4, 9]** Bir $x:\mathbb{N}\to\mathbb{R}$ fonksiyonu için $\Delta$ fark operatörü veya $x$' in birinci basamaktan farkı

$$\Delta x(n)=x(n+1)-x(n)$$

şeklinde tanımlanır.

**Tanım 2.2.3. [4, 9]** $n\geq n_0$ için $\Delta F(n)=f(n)$ olsun. Bu durumda $n\geq n_0$ için

$$\Delta^{-1}f(n)=F(n)+c$$

şeklinde tanımlanan $\Delta^{-1}$ operatörüne ters fark operatörü denir ve $F(n)$ fonksiyonuna da $f(n)$ ' in ters farkı denir. Burada *c* keyfi sabittir.

**Teorem 2.2.4. [4, 9]** Bir $f(n)$ fonksiyonu $n\geq n_0$ için tanımlı ise bu durumda

$$\Delta^{-1}f(n)=c+\sum_{i=n_0}^{n-1}f(i)$$

dir. Burada c keyfi bir sabittir.

**Tanım 2.2.5. [4, 9]** $n\in\mathbb{N}_0$ bağımsız değişken ve $x$ bilinmeyen fonksiyon olmak üzere

$$F\left(n,x(n),x(n+1),...,x(n+k)\right)=0,$$

eşitliğine fark denklemi denir.

Fark denklem literatüründe $x(n)$ yerine $x_n$ sembolü kullanılabilmektedir.

**Tanım 2.2.6. [4, 9]** Bir fark denkleminin mertebesi, denklemdeki en büyük indis ile en küçük indis arasındaki fark olarak tanımlanır. Örneğin;

$$x_{n+2}-2x_{n+1}+x_n=0,$$

denklemi ikinci mertebeden bir fark denklemidir.

**Tanım 2.2.7. [4, 9]** $a_1(n),\ a_2(n),\ldots,a_k(n)$ katsayıları ile $g(n),\ n\geq n_0$ için tanımlı reel değerli fonksiyonlar ve $\mathbb{N}_{n_0}$ üzerinde $a_k(n)\neq 0$ olmak üzere

$$x_{n+k}+a_1(n)x_{n+k-1}+...+a_k(n)x_n=g(n) \qquad (2.2.1)$$

biçimindeki denkleme lineer fark denklemi denir. Aksi haldeki fark denklemine lineer olmayan fark denklemi denir.

Lineer fark denklemleri katsayıları ve $g(n)$ ‘in durumuna göre sınıflandırılırlar.

i. Eğer $x_{n+k}+a_1(n)x_{n+k-1}+\ldots+a_k(n)x_n=g(n)$ denkleminde $g(n)=0$ ise denkleme *Lineer Homojen Fark Denklemi* denir.

ii. $\forall i\in\{1,2,...,k\}$ için $a_i(n)=a_i$ şeklinde katsayıları sabit iseler, denkleme *Sabit Katsayılı Lineer Fark Denklemi* denir.

iii. $\exists i\in\{1,2,...,k\}$ için $a_i(n)$ katsayılarından en az biri bağımsız değişkenin fonksiyonu ise denkleme *Değişken Katsayılı Lineer Fark Denklemi* denir.

**Tanım 2.2.8. [28]** $F_0=0,\ F_1=1$ ve

$$F_{n+2}=F_{n+1}+F_n,\ n\in\mathbb{N}_0, \quad (2.2.2)$$

ile tanımlanan $\{F_n\}_{n=0}^{\infty}$ dizisine Fibonacci dizisi denir. *n*. Fibonacci sayısı için Binet formülü ise

$$F_n=\frac{\lambda_1^n-\lambda_2^n}{\lambda_1-\lambda_2} \quad (2.2.3)$$

Burada $\lambda_{1,2}=\frac{1\pm\sqrt{5}}{2}$’dir.

**Tanım 2.2.9. [28]** $s_0=0,\ s_1=1$ başlangıç koşulları ve $\alpha,\beta\in\mathbb{R}$ için

$$s_{n+2}=\alpha s_{n+1}+\beta s_n,\ n\in\mathbb{N}_0, \quad (2.2.4)$$

ile tanımlanan $\{s_n\}_{n=0}^{\infty}$ dizisine genelleştirilmiş Fibonacci dizisi denir. *n*. Fibonacci sayısı için Binet formülü ise

$$s_n=\frac{\lambda_1^n-\lambda_2^n}{\lambda_1-\lambda_2},n\in\mathbb{N}_0, \quad (2.2.5)$$

Burada $\lambda_{1,2}=\frac{\alpha\pm\sqrt{\alpha^2+4\beta}}{2}$’dir.

Literatürde çok fazla karşılaşılan başlangıç koşulu $x_0$ olan

$$x_{n+1} = a_n x_n + b_n,\ n \in \mathbb{N}_0, \tag{2.2.6}$$

birinci mertebeden değişken katsayılı homojen olmayan lineer fark denkleminin genel çözümü

$$x_n = x_0 \prod_{k=0}^{n-1} a_k + \sum_{i=0}^{n-1} b_i \prod_{k=i+1}^{n-1} a_k \tag{2.2.7}$$

şeklindedir. Denklem (2.2.6)' da $\forall n \in \mathbb{N}_0$ için $a_n = 0$ ise denklem (2.2.6),

$$x_{n+1} = b_n,\ n \in \mathbb{N}_0, \tag{2.2.8}$$

formuna indirgenir. Bu durumda, denklem (2.2.8) genel çözümü,

$$x_n = b_{n-1},\ n \in \mathbb{N}_0, \tag{2.2.9}$$

bulunur. Denklem (2.2.6)' de $\forall n \in \mathbb{N}_0$ için $b_n = 0$ ise denklem (2.2.6) ,

$$x_{n+1} = a_n x_n,\ n \in \mathbb{N}_0, \tag{2.2.10}$$

değişken katsayılı birinci mertebeden homojen lineer fark denklemine dönüşür. Denklem (2.2.10)'in genel çözümü ise

$$x_n = x_0 \prod_{k=0}^{n-1} a_k \tag{2.2.11}$$

şeklindedir. Öte yandan $\forall n \in \mathbb{N}_0$ için $(a_n)_{n \in \mathbb{N}_o}$ ve $(b_n)_{n \in \mathbb{N}_o}$ dizileri sabit olduğu durumda, yani, $a_n = a,\ b_n = b,\ n \in \mathbb{N}_0,$ Denklem (2.2.6) birinci mertebeden sabit katsayılı homojen olmayan lineer fark denklemi olarak bilinen

$$x_{n+1} = a x_n + b,\ n \in \mathbb{N}_0 \tag{2.2.12}$$

fark denklemine indirgenir. Denklem (2.2.12)' nin genel çözümü ise

$$x_n = \begin{cases} a^n x_0 + b\dfrac{1-a^n}{1-a}, & eğer\ a \neq 1, \\ x_0 + bn, & eğer\ a = 1, \end{cases} \tag{2.2.13}$$

şeklindedir. Öte yandan, $a \in \mathbb{R} \setminus \{0\},\ b \in \mathbb{R}$ olmak üzere Denklem (2.2.12) ‘ yi alt dizi kabul eden $x_{kn+i} = a x_{k(n-1)+i} + b$ fark denkleminin genel çözümü ise

$$x_{kn+i}=\begin{cases}a^{n+1}x_{i-k}+b\dfrac{1-a^{n+1}}{1-a}, & a\neq 1\\ x_{i-k}+(n+1)b, & a=1\end{cases} \tag{2.2.14}$$

şeklinde ifade edilir.

Benzer şekilde literatürde çok fazla karşılaşılan, başlangıç koşulları $x_{-1},\ x_0$ olan ikinci mertebeden sabit katsayılı homojen lineer fark denklemi ise

$$x_{n+1}=ax_n+bx_{n-1},\ n\in\mathbb{N}_0, \tag{2.2.15}$$

şeklindedir. Bu denkleme ait $P(\lambda)=\lambda^2-a\lambda-b=0$ karakteristik denkleminin kökleri $\lambda_{1,2}=\dfrac{a\pm\sqrt{a^2+4b}}{2}$ olmak üzere, (2.2.15) denkleminin genel çözümü, $n\geq -1$ için,

$$x_n=\begin{cases}\dfrac{(\lambda_2x_{-1}-x_0)\lambda_1^{n+1}+(x_0-\lambda_1x_{-1})\lambda_2^{n+1}}{\lambda_2-\lambda_1}, & eğer\ a^2+4b\neq 0,\\ \left(x_0(n+1)-x_{-1}\lambda_1 n\right)\lambda_1^n, & eğer\ a^2+4b=0,\end{cases} \tag{2.2.16}$$

şeklindedir.

**Tanım 2.2.10. [5] (Genel Riccati Türü Denklemler)** $n\geq n_0$ için $c\neq 0$ ve $ad-bc\neq 0$ olmak üzere

$$x_{n+1}=\frac{ax_n+b}{cx_n+d} \tag{2.2.17}$$

denklemi *Genel Riccati türü* bir denklem olarak bilinir. Denklem (2.2.17)'nın bazı özel durumları aşağıdaki gibidir:

- Eğer $c=0$ ise Denklem (2.2.17)'nın çözümü lineer fark denklemidir.
- Eğer $c\neq 0$ ve $ad-bc=0$ ise Denklem (2.2.17)'nın çözümü $x_{n+1}=\dfrac{b}{d}$ şeklinde bir sabittir.
- Eğer $b+c=0$ ve $x_0\neq -\dfrac{d}{c}$ ise $\{x_n\}_{n=0}^{\infty}$ iken Denklem (2.2.17)'in çözümü iki periyotludur.

Öte yandan (2.2.17) denklemine

$$cx_n + d = \frac{y_{n+1}}{y_n},\ n \in \mathbb{N}_0, \tag{2.2.18}$$

dönüşümü uygulanırsa $y_{n+2} - (a+d)y_{n+1} - (bc - ad)y_n = 0,\ n \in \mathbb{N}_0$, ikinci mertebeden lineer fark denklemine indirgenir. Bu denklem $y_0$ ve $y_1$ başlangıç koşulları olmak üzere genel çözümü

$$y_n = \begin{cases} \dfrac{(\lambda_2 y_0 - y_1)\lambda_1^n + (y_1 - \lambda_1 y_0)\lambda_2^n}{\lambda_2 - \lambda_1}, & eğer\ \lambda_1 \neq \lambda_2, \\ \left(y_1(n+1) - y_0\lambda_1 n\right)\lambda_1^n, & eğer\ \lambda_1 = \lambda_2, \end{cases} \tag{2.2.19}$$

şeklindedir. $cx_0 + d = \dfrac{y_1}{y_0}$ ve (2.2.18) dönüşümünden $(x_n)_{n\in\mathbb{N}_0}$ dizisinin genel çözümü ise

$$x_n = \begin{cases} \dfrac{1}{c}\dfrac{\left(\lambda_2 - (cx_0+d)\right)\lambda_1^{n+1} + \left((cx_0+d) - \lambda_1\right)\lambda_2^{n+1}}{\left(\lambda_2 - (cx_0+d)\right)\lambda_1^n + \left((cx_0+d) - \lambda_1\right)\lambda_2^n} - \dfrac{d}{c}, & eğer\ \lambda_1 \neq \lambda_2, \\ \dfrac{1}{c}\dfrac{\left((cx_0+d)(n+2) - \lambda_1(n+1)\right)\lambda_1^{n+1}}{\left((cx_0+d)(n+1) - \lambda_1 n\right)\lambda_1^n} - \dfrac{d}{c}, & eğer\ \lambda_1 = \lambda_2, \end{cases} \tag{2.2.20}$$

elde edilir.

**Tanım 2.2.11 [9]** $\{x_n\}_{n=-k}^{\infty}$, $x_{n+1} = f(x_n, x_{n-1}, \ldots, x_{n-k}),\ n \in \mathbb{N}_0$, fark denkleminin bir çözümü olsun. Eğer $\{x_n\}_{n=-k}^{\infty}$ çözümü $n \geq -k$ için $x_{n+p} = x_n$ şartını sağlıyorsa $\{x_n\}_{n=-k}^{\infty}$ çözümü *p*-periyotludur denir. Bu şartı sağlayan en küçük pozitif *p* sayısına da asal periyot denir.

# 3. BÖLÜM

## 3.1 $x_{n+1}=\alpha y_n+\frac{ay_n}{y_n-\beta z_{n-1}},\ y_{n+1}=\beta z_n+\frac{bz_n}{z_n-\gamma x_{n-1}},\ z_{n+1}=\gamma x_n+\frac{cx_n}{x_n-\alpha y_{n-1}}$ Fark Denklem Sisteminin Çözümü

Bu bölümde aşağıda verilen ikinci mertebeden üç boyutlu fark denklem sisteminin

$$x_{n+1}=\alpha y_n+\frac{ay_n}{y_n-\beta z_{n-1}},\ y_{n+1}=\beta z_n+\frac{bz_n}{z_n-\gamma x_{n-1}},\ z_{n+1}=\gamma x_n+\frac{cx_n}{x_n-\alpha y_{n-1}},\quad n\in\mathbb{N}_0, \tag{3.1.1}$$

katsayılarının durumuna göre 10 alt durumda çözümleri elde edilecektir. Burada $a, b, c, \alpha, \beta, \gamma$ parametreleri ve $x_{-i},\ y_{-i},\ z_{-i}$, $i\in\{0,1\}$, başlangıç değerleri reel sabitlerdir.

Yukarıda verilen üç-boyutlu ikinci mertebeden lineer olmayan fark denklem sistemi, bu tezin bir parçası olan ve SCI-Exp. kapsamındaki "Mathematica Slovaca" isimli dergide kabul edilen ve 2023'de yayımlanacak olan makalemiz ile literatüre girecektir.

### 3.1.1 $a=0,\ bc\alpha\beta\gamma\neq 0$ durumu

Bu alt bölümde $a=0,\ bc\alpha\beta\gamma\neq 0$ durumunda (3.1.1) sistemi

$$x_{n+1}=\alpha y_n,\ y_{n+1}=\beta z_n+\frac{bz_n}{z_n-\gamma x_{n-1}},\ z_{n+1}=\gamma x_n+\frac{cx_n}{x_n-\alpha y_{n-1}},\ n\in\mathbb{N}_0, \tag{3.1.2}$$

sistemine indirgenir. $n=0$ için $x_1=\alpha y_0$ ve $n=1$ için $z_2=\gamma x_1+\frac{cx_1}{x_1-\alpha y_0}=\gamma x_1+\frac{cx_1}{0}$ olup $z_2$ nin iyi tanımlı olmadığı kolayca görülür.

### 3.1.2 $b=0,\ ac\alpha\beta\gamma\neq 0$ durumu

Bu alt bölümde $b=0,\ ac\alpha\beta\gamma\neq 0$ durumunda (3.1.1) sistemi

$$x_{n+1}=\alpha y_n+\frac{ay_n}{y_n-\beta z_{n-1}},\ y_{n+1}=\beta z_n,\ z_{n+1}=\gamma x_n+\frac{cx_n}{x_n-\alpha y_{n-1}},\ n\in\mathbb{N}_0, \tag{3.1.3}$$

sistemine indirgenir. $n=0$ için $y_1=\beta z_0$ ve $n=1$ için $x_2=\alpha y_1+\frac{ay_1}{y_1-\beta z_0}=\alpha y_1+\frac{ay_1}{0}$ olup

$x_2$'nin iyi tanımlı olmadığı kolayca görülür.

### 3.1.3 $c=0,\ ab\alpha\beta\gamma\neq 0$ durumu

Bu alt bölümde $c=0,\ ab\alpha\beta\gamma\neq 0$ durumunda (3.1.1) sistemi

$$x_{n+1}=\alpha y_n+\frac{ay_n}{y_n-\beta z_{n-1}},\ y_{n+1}=\beta z_n+\frac{bz_n}{z_n-\gamma x_{n-1}},\ z_{n+1}=\gamma x_n,\ n\in\mathbb{N}_0, \tag{3.1.4}$$

sistemine indirgenir. $n=0$ için $z_1=\gamma x_0$ ve $n=1$ için $y_2=\beta z_1+\frac{bz_1}{z_1-\gamma x_0}=\beta z_1+\frac{bz_1}{0}$ olup $y_2$ 'nin iyi tanımlı olmadığı kolayca görülür.

### 3.1.4 $\alpha=0,\ abc\beta\gamma\neq 0$ durumu

Bu alt bölümde $\alpha=0,\ abc\beta\gamma\neq 0$ durumunda (3.1.1) sistemi

$$x_{n+1}=\frac{ay_n}{y_n-\beta z_{n-1}},\ y_{n+1}=\beta z_n+\frac{bz_n}{z_n-\gamma x_{n-1}},\ z_{n+1}=\gamma x_n+c,\ n\in\mathbb{N}_0, \tag{3.1.5}$$

sistemine indirgenir. Sistem (3.1.5) ‘ teki üçüncü denklem, ikinci denklemde, daha sonra da ikinci denklemde elde edilen ifade ve üçüncü denklem birinci denklemde yerine yazılırsa

$$\begin{aligned}
x_{n+1}&=\frac{\left(a\beta\gamma+\frac{ab\gamma}{c}\right)x_{n-2}+ac\beta+ab}{\left(\beta\gamma+\frac{b\gamma}{c}\right)x_{n-2}+\beta c+b-\beta\gamma x_{n-2}-\beta c}\\
&=\frac{\left(a\beta\gamma+\frac{ab\gamma}{c}\right)x_{n-2}+ac\beta+ab}{\frac{b\gamma}{c}x_{n-2}+b}\\
&=\frac{\left(a\beta\gamma+\frac{ab\gamma}{c}\right)x_{n-2}+ac\beta+ab}{\frac{b\gamma}{c}x_{n-2}+b}\\
&=a+\frac{ac\beta}{b},\ n\geq 2,
\end{aligned} \tag{3.1.6}$$

elde edilir. (3.1.6)’ dan $\{x_n\}_{n\geq 3}$ çözümü $x_n=a+\frac{ac\beta}{b}$ şeklinde elde edilir. Öte yandan elde edilen bu çözüm, Sistem (3.1.5) ‘ teki üçüncü denklemde yerine yazılırsa $\{z_n\}_{n\geq 4}$ dizisinin çözümü

$$z_n=\gamma\left(a+\frac{ac\beta}{b}\right)+c,\ n\geq 4,$$

bulunur. Son olarak Sistem (3.1.5)' te $\{x_n\}$ ve $\{z_n\}$ çözümleri ikinci denklemde yerine yazılırsa

$$\begin{aligned} y_{n+1} &= \beta z_n + \frac{bz_n}{z_n - \gamma x_{n-1}} \\ &= \beta(\gamma x_{n-1} + \mathrm{c}) + \frac{b(\gamma x_{n-1} + \mathrm{c})}{\gamma x_{n-1} + \mathrm{c} - \gamma x_{n-1}} \\ &= \left(\beta\gamma + \frac{b\gamma}{c}\right) x_{n-1} + \beta c + b, \ \ n \geq 1, \end{aligned} \tag{3.1.7}$$

elde edilir. (3.1.7) eşitliğinde $n$ yerine $n-1$ yazılırsa $\{y_n\}_{n\geq 5}$ çözümü

$$\begin{aligned} y_n &= \left(\beta\gamma + \frac{b\gamma}{c}\right) x_{n-1} + \beta c + b \\ &= \left(\beta\gamma + \frac{b\gamma}{c}\right)\left(a + \frac{ac\beta}{b}\right) + \beta c + b \\ &= \frac{(b + c\beta)(bc + ba\gamma + ac\beta\gamma)}{cb} \end{aligned} \tag{3.1.8}$$

elde edilir. Böylece Sistem (3.1.5)'in çözümü

$$\begin{cases} x_n = a + \dfrac{ac\beta}{b}, \ \ n \geq 3, \\ y_n = \dfrac{(b + c\beta)(bc + ba\gamma + ac\beta\gamma)}{cb}, \ \ n \geq 5, \\ z_n = a\gamma + c + \dfrac{ac\beta\gamma}{b}, \ \ n \geq 4, \end{cases} \tag{3.1.9}$$

olarak bulunur.

### 3.1.5 $\beta = 0, \ abc\alpha\gamma \neq 0$ durumu

Bu alt bölümde $\beta = 0, \ abc\alpha\gamma \neq 0$ durumunda (3.1.1) sistemi

$$x_{n+1} = \alpha y_n + a, \ y_{n+1} = \frac{bz_n}{z_n - \gamma x_{n-1}}, \ z_{n+1} = \gamma x_n + \frac{cx_n}{x_n - \alpha y_{n-1}}, \ n \in \mathbb{N}_0 \tag{3.1.10}$$

sistemine indirgenir. Sistem (3.1.10)'daki birinci denklem, üçüncü denklemde daha sonra da üçüncü denklemde elde edilen ifade ve birinci denklem ikinci denklemde yerine yazılırsa

$$y_{n+1}=\frac{\left(b\alpha\gamma+\frac{bc\alpha}{a}\right)y_{n-2}+ba\gamma+bc}{\left(\alpha\gamma+\frac{\alpha c}{a}\right)y_{n-2}+a\gamma+c-\alpha\gamma y_{n-2}-a\gamma}$$

$$=\frac{\left(b\alpha\gamma+\frac{bc\alpha}{a}\right)y_{n-2}+ba\gamma+bc}{\frac{\alpha c}{a}y_{n-2}+c} \tag{3.1.11}$$

$$=\left(\frac{ba\gamma+bc}{c}\right)\frac{\frac{\alpha c}{a}y_{n-2}+c}{\frac{\alpha c}{a}y_{n-2}+c}$$

$$=b+\frac{ba\gamma}{c},\ \ n\geq 2,$$

elde edilir. (3.1.11)'den $\{y_n\}_{n\geq 3}$ çözümü $y_n=b+\frac{ba\gamma}{c}$ şeklinde bulunur. Öte yandan elde edilen bu çözüm, Sistem (3.1.10)'daki birinci denklemde yerine yazılırsa $\{x_n\}_{n\geq 4}$ dizisinin çözümü

$$x_n=b\alpha+a+\frac{ab\alpha\gamma}{c},\ n\geq 4, \tag{3.1.12}$$

bulunur. Son olarak Sistem (3.1.10)'da $\{y_n\}$ ve $\{x_n\}$ çözümleri üçüncü denklemde yerine yazılırsa

$$z_{n+1}=\gamma x_n+\frac{cx_n}{x_n-\alpha y_{n-1}}$$

$$=\gamma\left(\alpha y_{n-1}+a\right)+\frac{c\left(\alpha y_{n-1}+a\right)}{\alpha y_{n-1}+a-\alpha y_{n-1}} \tag{3.1.13}$$

$$=\left(\alpha\gamma+\frac{\alpha c}{a}\right)y_{n-1}+a\gamma+c,\ n\geq 1,$$

elde edilir. (3.1.13) eşitliğinde $n$ yerine $n-1$ yazılırsa $\{z_n\}_{n\geq 5}$ çözümü

$$z_n=\left(\alpha\gamma+\frac{\alpha c}{a}\right)y_{n-1}+a\gamma+c$$

$$=\left(\alpha\gamma+\frac{\alpha c}{a}\right)\left(b+\frac{ba\gamma}{c}\right)+a\gamma+c \tag{3.1.14}$$

$$=\frac{(c+a\gamma)(ac+cb\alpha+ab\alpha\gamma)}{ac}$$

elde edilir. Böylece Sistem (3.1.10)'un çözümü

$$\begin{cases} x_n = b\alpha + a + \dfrac{ab\alpha\gamma}{c}, \ n \geq 4, \\ y_n = b + \dfrac{ba\gamma}{c}, \ n \geq 3, \\ z_n = \dfrac{(c + a\gamma)(ac + cb\alpha + ab\alpha\gamma)}{ac}, \ n \geq 5, \end{cases} \tag{3.1.15}$$

olarak bulunur.

### 3.1.6 $\gamma = 0,\ abc\alpha\beta \neq 0$ durumu

Bu alt bölümde $\gamma = 0,\ abc\alpha\beta \neq 0$ durumunda (3.1.1) sistemi

$$x_{n+1} = \alpha y_n + \frac{a y_n}{y_n - \beta z_{n-1}},\ y_{n+1} = \beta z_n + b,\ z_{n+1} = \frac{c x_n}{x_n - \alpha y_{n-1}},\ n \in \mathbb{N}_0, \tag{3.1.16}$$

sistemine indirgenir. Sistem (3.1.16)'te ikinci denklem, birinci denklemde, daha sonra da birinci denklemde elde edilen ifade üçüncü denklemde yerine yazılırsa

$$\begin{aligned} z_{n+1} &= \frac{\left(c\alpha\beta + \dfrac{ca\beta}{b}\right) z_{n-2} + cb\alpha + ca}{\left(\alpha\beta + \dfrac{a\beta}{b}\right) z_{n-2} + b\alpha + a - \alpha\beta z_{n-2} - b\alpha} \\ &= \frac{\left(c\alpha\beta + \dfrac{ca\beta}{b}\right) z_{n-2} + cb\alpha + ca}{\dfrac{a\beta}{b} z_{n-2} + a} \\ &= \left(\frac{ca + cb\alpha}{a}\right) \frac{\dfrac{a\beta}{b} z_{n-2} + a}{\dfrac{a\beta}{b} z_{n-2} + a} \\ &= c + \frac{cb\alpha}{a}, n \geq 2, \end{aligned} \tag{3.1.17}$$

elde edilir. (3.1.17)'dan $\{z_n\}_{n\geq 3}$ çözümü $z_n = c + \dfrac{cb\alpha}{a}$ şeklinde elde edilir. Öte yandan elde edilen bu çözüm, Sistem (3.1.16)'deki ikinci denklemde yerine yazılırsa $\{y_n\}_{n\geq 4}$ dizisinin çözümü

$$y_n = b + c\beta + \frac{cb\alpha\beta}{a} \tag{3.1.18}$$

bulunur. Son olarak Sistem (3.1.16)'de $\{y_n\}$ ve $\{z_n\}$ çözümleri birinci denklemde yerine yazılırsa

$$\begin{aligned} x_{n+1} &= \alpha y_n + \frac{a y_n}{y_n - \beta z_{n-1}} \\ &= \alpha\left(\beta z_{n-1} + b\right) + \frac{a\left(\beta z_{n-1} + b\right)}{\beta z_{n-1} + b - \beta z_{n-1}} \\ &= \left(\alpha\beta + \frac{a\beta}{b}\right) z_{n-1} + b\alpha + a, \ n \geq 1, \end{aligned} \tag{3.1.19}$$

elde edilir.(3.1.19) eşitliğinde $n$ yerine $n-1$ yazılırsa $\{x_n\}_{n\geq 5}$ çözümü

$$\begin{aligned} x_n &= \left(\alpha\beta + \frac{a\beta}{b}\right) z_{n-1} + b\alpha + a \\ &= \left(\alpha\beta + \frac{a\beta}{b}\right)\left(c + \frac{cb\alpha}{a}\right) + b\alpha + a \\ &= \frac{\left(a + b\alpha\right)\left(ab + ac\beta + bc\alpha\beta\right)}{ab} \end{aligned} \tag{3.1.20}$$

elde edilir. Böylece Sistem (3.1.16)'in çözümü

$$\begin{cases} x_n = \dfrac{\left(a + b\alpha\right)\left(ab + ac\beta + bc\alpha\beta\right)}{ab}, \ n \geq 5, \\ y_n = b + c\beta + \dfrac{cb\alpha\beta}{a}, \ n \geq 4, \\ z_n = c + \dfrac{cb\alpha}{a}, \ n \geq 3, \end{cases} \tag{3.1.21}$$

olarak bulunur.

### 3.1.7. $\alpha = 0, \ \beta = 0, \ abc\gamma \neq 0$ durumu

Bu alt bölümde $\alpha = 0, \beta = 0, abc\gamma \neq 0$ durumunda (3.1.1) denklem sistemi

$$x_{n+1} = a, \ y_{n+1} = \frac{bz_n}{z_n - \gamma x_{n-1}}, \ z_{n+1} = \gamma x_n + c, \ n \in \mathbb{N}_0, \tag{3.1.22}$$

sistemine indirgenir. (3.1.22) sisteminde birinci denklemde $n$ yerine $n-1$ yazılırsa $\{x_n\}_{n\geq 1}$ çözümü $x_n = a$ elde edilir ve üçüncü denklemde yerine yazılırsa $\{z_n\}_{n\geq 2}$ çözümü

$$\begin{aligned} z_n &= \gamma x_{n-1} + c \\ &= a\gamma + c \end{aligned} \tag{3.1.23}$$

bulunur. Öte yandan $\{x_n\}$ ve $\{z_n\}$ çözümleri ikinci denklemde yerine yazılırsa $\{y_n\}_{n\geq 3}$ çözümü

$$\begin{aligned} y_n &= \frac{bz_{n-1}}{z_{n-1} - \gamma x_{n-2}} \\ &= \frac{b(a\gamma + c)}{a\gamma + c - a\gamma} \\ &= \frac{ab\gamma}{c} + b \end{aligned} \tag{3.1.24}$$

bulunur. Böylece Sistem (3.1.22)'in çözümü

$$\begin{cases} x_n = a,\ n \geq 1, \\ y_n = \dfrac{ba\gamma}{c} + b,\ n \geq 3, \\ z_n = a\gamma + c,\ n \geq 2, \end{cases} \tag{3.1.25}$$

olarak bulunur.

### 3.1.8 $\alpha = 0,\ \gamma = 0,\ abc\beta \neq 0$ durumu

Bu alt bölümde $\alpha = 0,\ \gamma = 0,\ abc\beta \neq 0$ durumunda (3.1.1) sistemi

$$x_{n+1} = \frac{ay_n}{y_n - \beta z_{n-1}},\ y_{n+1} = \beta z_n + b,\ z_{n+1} = c,\ n \in \mathbb{N}_0, \tag{3.1.26}$$

sistemine indirgenir. Sistem (3.1.26)'te üçüncü denklemde $n$ yerine $n-1$ yazılırsa $\{z_n\}_{n\geq 1}$ çözümü $z_n = c$ elde edilir ve ikinci denklemde yerine yazılırsa $\{y_n\}_{n\geq 2}$ çözümü

$$\begin{aligned} y_n &= \beta z_{n-1} + b \\ &= \beta c + b \end{aligned} \tag{3.1.27}$$

bulunur. Öte yandan $\{y_n\}$ ve $\{z_n\}$ çözümleri birinci denklemde yerine yazılırsa $\{x_n\}_{n\geq 3}$ çözümü

$$\begin{aligned} x_n &= \frac{ay_{n-1}}{y_{n-1}-\beta z_{n-2}} \\ &= \frac{ac\beta}{b}+a \end{aligned} \tag{3.1.28}$$

bulunur. Böylece Sistem (3.1.26)'in çözümü

$$\begin{cases} x_n = \dfrac{ac\beta}{b}+a, \ n\geq 3, \\ y_n = c\beta + b, \ n\geq 2, \\ z_n = c, \ n\geq 1, \end{cases} \tag{3.1.29}$$

olarak elde edilir.

**3.1.9 $\beta=0,\ \gamma=0,\ abc\alpha\neq 0$ durumu**

Bu alt bölümde $\beta=0, \gamma=0, abc\alpha\neq 0$ durumunda (3.1.1) sistemi

$$x_{n+1}=\alpha y_n + a,\ y_{n+1}=b,\ z_{n+1}=\frac{cx_n}{x_n-\alpha y_{n-1}},\ n\in\mathbb{N}_0, \tag{3.1.30}$$

sistemine indirgenir. Sistem (3.1.30)'da ikinci denklemde $n$ yerine $n-1$ yazılırsa $\{y_n\}_{n\geq 1}$ çözümü $y_n=b$ elde edilir ve birinci denklemde yerine yazılırsa $\{x_n\}_{n\geq 2}$ çözümü

$$\begin{aligned} x_n &= \alpha y_{n-1}+a \\ &= b\alpha + a \end{aligned} \tag{3.1.31}$$

bulunur. Öte yandan $\{y_n\}$ ve $\{x_n\}$ çözümleri üçüncü denklemde yerine yazılırsa $\{z_n\}_{n\geq 3}$ çözümü

$$\begin{aligned} z_n &= \frac{cx_{n-1}}{x_{n-1}-\alpha y_{n-2}} \\ &= \frac{bc\alpha}{a}+c \end{aligned} \tag{3.1.32}$$

bulunur. Böylece Sistem (3.1.30)'in çözümü

$$\begin{cases} x_n = b\alpha + a, \ n \geq 2, \\ y_n = b, \ n \geq 1, \\ z_n = \dfrac{bc\alpha}{a} + c, \ n \geq 3, \end{cases} \tag{3.1.33}$$

olarak bulunur.

### 3.1.10 $abc\alpha\beta\gamma \neq 0$ durumu

Bu alt bölümde

$$x_{n+1} = \alpha y_n + \frac{ay_n}{y_n - \beta z_{n-1}}, \ y_{n+1} = \beta z_n + \frac{bz_n}{z_n - \gamma x_{n-1}}, \ z_{n+1} = \gamma x_n + \frac{cx_n}{x_n - \alpha y_{n-1}}, n \in \mathbb{N}_0, \tag{3.1.34}$$

sisteminde ilk olarak ikinci denklem birinci denklemde yerine yazılırsa

$$\begin{aligned} x_{n+1} - \alpha y_n &= \frac{ay_n}{y_n - \beta z_{n-1}} \\ &= \frac{ay_n}{\dfrac{bz_{n-1}}{z_{n-1} - \gamma x_{n-2}}} \\ &= \frac{ay_n z_{n-1} - a\gamma y_n x_{n-2}}{bz_{n-1}} \end{aligned} \tag{3.1.35}$$

elde edilir. Denklem (3.1.35)'ün her iki tarafı $y_n$'e bölünür ve gerekli işlemler yapılırsa

$$\frac{x_{n+1}}{y_n} = \alpha + \frac{a}{b} - \frac{a\gamma}{b} \frac{x_{n-2}}{z_{n-1}}, \ n \in \mathbb{N}, \tag{3.1.36}$$

bulunur. İkinci olarak üçüncü denklem ikinci denklemde yerine yazılırsa

$$\begin{aligned} y_{n+1} - \beta z_n &= \frac{bz_n}{z_n - \gamma x_{n-1}} \\ &= \frac{bz_n}{\dfrac{cx_{n-1}}{x_{n-1} - \alpha y_{n-2}}} \\ &= \frac{bz_n x_{n-1} - b\alpha z_n y_{n-2}}{cx_{n-1}} \end{aligned} \tag{3.1.37}$$

elde edilir. Denklem (3.1.37)'nın her iki tarafı $z_n$'e bölünür ve gerekli işlemler yapılırsa

$$\frac{y_{n+1}}{z_n} = \beta + \frac{b}{c} - \frac{b\alpha}{c}\frac{y_{n-2}}{x_{n-1}},\ n \in \mathbb{N}, \tag{3.1.38}$$

bulunur. Son olarak üçüncü denklemde birinci denklem yerine yazılırsa

$$\begin{aligned} z_{n+1} - \gamma x_n &= \frac{cx_n}{x_n - \alpha y_{n-1}} \\ &= \frac{cx_n}{\dfrac{ay_{n-1}}{y_{n-1} - \beta z_{n-2}}} \\ &= \frac{cx_n y_{n-1} - c\beta x_n z_{n-2}}{ay_{n-1}} \end{aligned} \tag{3.1.39}$$

elde edilir. Denklem (3.1.39)'in her iki tarafı $x_n$ 'e bölünür ve gerekli işlemler yapılırsa

$$\frac{z_{n+1}}{x_n} = \gamma + \frac{c}{a} - \frac{c\beta}{a}\frac{z_{n-2}}{y_{n-1}},\ n \in \mathbb{N}, \tag{3.1.40}$$

bulunur. Böylece (3.1.35), (3.1.37), (3.1.39) sistemlerinde sırasıyla $\frac{x_n}{y_{n-1}} = u_n,\ \frac{y_n}{z_{n-1}} = v_n,\ \frac{z_n}{x_{n-1}} = w_n,\ n \in \mathbb{N}_0,$ dönüşümü uygulanırsa

$$\begin{aligned} u_{n+1} &= \alpha + \frac{a}{b} - \frac{a\gamma}{b}\frac{1}{w_{n-1}},\ n \in \mathbb{N}, \\ v_{n+1} &= \beta + \frac{b}{c} - \frac{b\alpha}{c}\frac{1}{u_{n-1}},\ n \in \mathbb{N}, \\ w_{n+1} &= \gamma + \frac{c}{a} - \frac{c\beta}{a}\frac{1}{v_{n-1}},\ n \in \mathbb{N}, \end{aligned} \tag{3.1.41}$$

bulunur. Sistem (3.1.41)'te üçüncü denklem, birinci denklemde yerine yazılırsa ve elde edilen ifadede ikinci denklemde yerine yazılırsa

$$
u_{n+1}=\left(\frac{\alpha b+a}{b}\right)-\frac{a\gamma}{b\left[\left(\frac{a\gamma+c}{a}\right)-\frac{c\beta}{a\left(\frac{(c\beta+b)u_{n-5}-b\alpha}{cu_{n-5}}\right)}\right]}
$$

$$
=\left(\frac{\alpha b+a}{b}\right)-\frac{a\gamma}{b\left[\frac{(ac\beta\gamma+ab\gamma+bc)u_{n-5}-ab\alpha\gamma-bc\alpha}{a\left((c\beta+b)u_{n-5}-b\alpha\right)}\right]}
$$

$$
=\left(\frac{(\alpha b+a)\left((ac\beta\gamma+ab\gamma+bc)u_{n-5}-ab\alpha\gamma-bc\alpha\right)-\left((a^2c\beta\gamma+a^2b\gamma)u_{n-5}-a^2b\alpha\gamma\right)}{b\left((ac\beta\gamma+ab\gamma+bc)u_{n-5}-ab\alpha\gamma-bc\alpha\right)}\right) \quad (3.1.42)
$$

$$
=\frac{b\left[(ac\alpha\beta\gamma+ab\alpha\gamma+cb\alpha+ac)u_{n-5}-ab\alpha^2\gamma-bc\alpha^2-ac\alpha\right]}{b\left[(ac\beta\gamma+ab\gamma+cb)u_{n-5}-ab\alpha\gamma-bc\alpha\right]}
$$

$$
=\frac{(ac\alpha\beta\gamma+ab\alpha\gamma+bc\alpha+ac)u_{n-5}-(ab\alpha^2\gamma+bc\alpha^2+ac\alpha)}{(ac\beta\gamma+ab\gamma+bc)u_{n-5}-(ab\alpha\gamma+bc\alpha)},\ n\geq 5,
$$

Riccati tipi fark denklemi haline dönüşür. Denklem (3.1.42)'de $n$ yerine $n\to 6n+i$ değişken değişimi yapılırsa (3.1.42) denkleminde $i\in\{0,1,2,3,4,5\}$ için

$$
u_{6n+i}=\frac{(ac\alpha\beta\gamma+ab\alpha\gamma+bc\alpha+ac)u_{6(n-1)+i}-(ab\alpha^2\gamma+bc\alpha^2+ac\alpha)}{(ac\beta\gamma+ab\gamma+bc)u_{6(n-1)+i}-(ab\alpha\gamma+bc\alpha)},\ n\geq 1, \quad (3.1.43)
$$

şeklinde yazılabilir. Denklem (3.1.43)' de $i\in\{0,1,2,3,4,5\}$ için $u_{6n+i}=u_n^{(i)}$ dönüşümü uygulanırsa

$$
u_n^{(i)}=\frac{(ac\alpha\beta\gamma+ab\alpha\gamma+bc\alpha+ac)u_{n-1}^{(i)}-(ab\alpha^2\gamma+bc\alpha^2+ac\alpha)}{(ac\beta\gamma+ab\gamma+bc)u_{n-1}^{(i)}-(ab\alpha\gamma+bc\alpha)},\ n\geq 1, \quad (3.1.44)
$$

elde edilir. İşlemlerin daha kolay anlaşılabilmesi adına $n\in\mathbb{N}_0$ için $u_n^{(i)}=\hat{u}_n$ dönüşümü tanımlansın. Bu durumda $u_0^{(i)}=\hat{u}_0$ olacağı açıktır. Buradan Denklem (3.1.44)

$$
\hat{u}_{n+1}=\frac{(ac\alpha\beta\gamma+ab\alpha\gamma+bc\alpha+ac)\hat{u}_n-(ab\alpha^2\gamma+bc\alpha^2+ac\alpha)}{(ac\beta\gamma+ab\gamma+bc)\hat{u}_n-(ab\alpha\gamma+bc\alpha)},\ n\in\mathbb{N}_0, \quad (3.1.45)
$$

iyi bilinen Riccati fark denklemine indirgenir. Denklem (3.1.45)' de $n \in \mathbb{N}_0$ için $\left(ac\beta\gamma + ab\gamma + bc\right)\hat{u}_n - ab\alpha\gamma + bc\alpha = \dfrac{p_{n+1}}{p_n}$ dönüşümü uygulanırsa

$$p_{n+2} = \left(ac\alpha\beta\gamma + ac\right) p_{n+1} - \left(a^2c^2\alpha\beta\gamma\right) p_n,\ n \in \mathbb{N}_0, \tag{3.1.46}$$

2. mertebeden sabit katsayılı lineer fark denklemine indirgenir. Denklem (3.1.46)' a ait karakteristik denklem $\lambda^2 - \left(ac\alpha\beta\gamma + ac\right)\lambda + a^2c^2\alpha\beta\gamma = 0$ şeklindedir. Karakteristik denklemin kökleri farklı ise, yani $\left(ac\alpha\beta\gamma + ac\right)^2 - 4\left(a^2c^2\alpha\beta\gamma\right) \neq 0$, kökler $\lambda_1 = ac\alpha\beta\gamma$ ve $\lambda_2 = ac$ olur. Buradan $p_0$ ve $p_1$ Denklem (3.1.46)' ın başlangıç şartları olmak üzere verilen başlangıç değer probleminin çözümü

$$p_n = \frac{\left(p_1 - p_0\lambda_2\right)\lambda_1^{\,n}}{\lambda_1 - \lambda_2} - \frac{\left(p_1 - p_0\lambda_1\right)\lambda_2^{\,n}}{\lambda_1 - \lambda_2}, n \in \mathbb{N}_0, \tag{3.1.47}$$

elde edilir. Öte yandan $\left(ac\beta\gamma + ab\gamma + bc\right)\hat{u}_n - ab\alpha\gamma + bc\alpha = \dfrac{p_{n+1}}{p_n}$ dönüşümünden

$$\begin{aligned}\left(ac\beta\gamma + ab\gamma + bc\right)\hat{u}_n - ab\alpha\gamma + bc\alpha &= \frac{\left(p_1 - p_0\lambda_2\right)\lambda_1^{\,n+1} - \left(p_1 - p_0\lambda_1\right)\lambda_2^{\,n+1}}{\left(p_1 - p_0\lambda_2\right)\lambda_1^{\,n} - \left(p_1 - p_0\lambda_1\right)\lambda_2^{\,n}} \\ &= \frac{\left(\frac{p_1}{p_0} - \lambda_2\right)\lambda_1^{\,n+1} - \left(\frac{p_1}{p_0} - \lambda_1\right)\lambda_2^{\,n+1}}{\left(\frac{p_1}{p_0} - \lambda_2\right)\lambda_1^{\,n} - \left(\frac{p_1}{p_0} - \lambda_1\right)\lambda_2^{\,n}}\end{aligned} \tag{3.1.48}$$

buluunur. Şimdi karmaşıklığı önlemek için $X_1 = ac\beta\gamma + ab\gamma + bc,\ Y_1 = ab\alpha\gamma + bc\alpha$ ve $Z_1 = a^2c^2\alpha\beta\gamma$ olmak üzere Denklem (3.1.48)' den

$$\hat{u}_n = \frac{1}{X_1}\frac{\left(X_1\hat{u}_0 - Y_1\right)\left(\dfrac{\lambda_1^{\,n+1} - \lambda_2^{\,n+1}}{\lambda_1 - \lambda_2}\right) - \lambda_1\lambda_2\left(\dfrac{\lambda_1^{\,n} - \lambda_2^{\,n}}{\lambda_1 - \lambda_2}\right)}{\left(X_1\hat{u}_0 - Y_1\right)\left(\dfrac{\lambda_1^{\,n} - \lambda_2^{\,n}}{\lambda_1 - \lambda_2}\right) - \lambda_1\lambda_2\left(\dfrac{\lambda_1^{\,n-1} - \lambda_2^{\,n-1}}{\lambda_1 - \lambda_2}\right)} + \frac{Y_1}{X_1},\ n \in \mathbb{N}_0, \tag{3.1.49}$$

elde edilir. $u_{6n+i} = u_n^{(i)} = \hat{u}_n$ değişken değişimleri ve Tanım 2.2.10. 'dan

$$u_{6n+i} = \frac{1}{X_1}\frac{\left(X_1u_i - Y_1\right)s_{n+1} - Z_1s_n}{\left(X_1u_i - Y_1\right)s_n - Z_1\gamma s_{n-1}} + \frac{Y_1}{X_1},\ n \in \mathbb{N}_0. \tag{3.1.50}$$

Burada $i=\{0,1,2,3,4,5\}$'dir. Diğer taraftan $\left(ac\alpha\beta\gamma+ac\right)^2-4\left(a^2c^2\alpha\beta\gamma\right)=0$ ise, yani karakteristik denklemin kökleri çakışık ise $\lambda_1=\lambda_2=\dfrac{ac\alpha\beta\gamma+ac}{2}$ olup, Denklem (3.1.46)' nın çözümü

$$p_n=c_1\lambda_1^{\,n}+c_2n\lambda_1^{\,n},n\in\mathbb{N}_0, \tag{3.1.51}$$

şeklindedir. $p_0$ ve $p_1$ Denklem (3.1.51)'in başlangıç şartları olmak üzere bu koşullar altındaki başlangıç değer probleminin çözümü

$$p_n=p_0\lambda_1^{\,n}+\left(p_1-p_0\lambda_1\right)n\lambda_1^{\,n-1},\ n\in\mathbb{N}_0, \tag{3.1.52}$$

dır. Benzer şekilde $\left(ac\beta\gamma+ab\gamma+bc\right)\hat{u}_n-\ ab\alpha\gamma+bc\alpha=\dfrac{p_{n+1}}{p_n}$ dönüşümünden

$$\left(ac\beta\gamma+ab\gamma+bc\right)\hat{u}_n-\ ab\alpha\gamma+bc\alpha=\frac{p_0\lambda_1^{\,n+1}+\left(p_1-p_0\lambda_1\right)\left(n+1\right)\lambda_1^{\,n}}{p_0\lambda_1^{\,n}+\left(p_1-p_0\lambda_1\right)n\lambda_1^{\,n-1}} \tag{3.1.53}$$

bulunur. Şimdi $X_1=ac\beta\gamma+ab\gamma+bc,\ Y_1=ab\alpha\gamma+bc\alpha$ ifadeleri dikkate alınırsa Denklem (3.1.53)' den

$$\hat{u}_n=\frac{1}{X_1}\frac{\left(X_1\hat{u}_0-Y_1\right)\left(n+1\right)\lambda_1^{\,n}-n\lambda_1^{\,n+1}}{\left(X_1\hat{u}_0-Y_1\right)n\lambda_1^{\,n-1}-\left(n-1\right)\lambda_1^{\,n}}+\frac{Y_1}{X_1},n\in\mathbb{N}_0, \tag{3.1.54}$$

elde edilir. $u_{6n+i}=u_n^{(i)}=\hat{u}_n$ değişken değişimlerinden

$$u_{6n+i}=\frac{1}{X_1}\frac{\left(X_1u_i-Y_1\right)\left(n+1\right)\lambda_1^{\,n}-n\lambda_1^{\,n+1}}{\left(X_1u_i-Y_1\right)n\lambda_1^{\,n-1}-\left(n-1\right)\lambda_1^{\,n}}+\frac{Y_1}{X_1},\ n\in\mathbb{N}_0, \tag{3.1.55}$$

elde edilir. Burada $i=\{0,1,2,3,4,5\}$'dir. Sistem (3.1.41)'de birinci denklem, ikinci denklemde yerine yazılırsa ve elde edilen ifadede üçüncü denklem yerine yazılırsa

$$v_{n+1}=\left(\beta+\frac{b}{c}\right)-\frac{b\alpha}{c\left[\left(\alpha+\frac{a}{b}\right)-\frac{a\gamma}{b\left(\left(\gamma+\frac{c}{a}\right)-\frac{c\beta}{av_{n-5}}\right)}\right]}$$

$$=\left(\frac{c\beta+b}{c}\right)-\frac{b\alpha}{c\left[\frac{\left(ab\alpha\gamma+cb\alpha+ca\right)v_{n-5}-cb\alpha\beta-ca\beta}{b\left[\left(a\gamma+c\right)v_{n-5}-c\beta\right]}\right]}$$

$$=\left(\frac{\left(c\beta+b\right)\left(\left(ab\alpha\gamma+cb\alpha+ca\right)v_{n-5}-cb\alpha\beta-ca\beta\right)-\left((ab^2\alpha\gamma+cb^2\alpha)v_{n-5}-cb^2\alpha\beta\right)}{c\left(\left(ab\alpha\gamma+cb\alpha+ca\right)v_{n-5}-cb\alpha\beta-ca\beta\right)}\right) \quad (3.1.56)$$

$$=\frac{c\left[\left(ab\alpha\beta\gamma+cb\alpha\beta+ca\beta+ab\right)v_{n-5}-\left(cb\alpha\beta^2+ca\beta^2+ab\beta\right)\right]}{c\left[\left(ab\alpha\gamma+cb\alpha+ac\right)v_{n-5}-\left(cb\alpha\beta+ca\beta\right)\right]}$$

$$=\frac{\left(ab\alpha\beta\gamma+cb\alpha\beta+ca\beta+ab\right)v_{n-5}-\left(cb\alpha\beta^2+ca\beta^2+ab\beta\right)}{\left(ab\alpha\gamma+cb\alpha+ac\right)v_{n-5}-\left(cb\alpha\beta+ca\beta\right)}, n\geq 5,$$

Riccati fark denklemi haline gelir. Denklem (3.1.56)’ da n yerine $n\to 6n+i$ değişken değişimi yapılırsa (3.1.56) denkleminde $i\in\{0,1,2,3,4,5\}$ için

$$v_{6n+i}=\frac{\left(ab\alpha\beta\gamma+cb\alpha\beta+ca\beta+ab\right)v_{6(n-1)+i}-\left(cb\alpha\beta^2+ca\beta^2+ab\beta\right)}{\left(ab\alpha\gamma+cb\alpha+ac\right)v_{6(n-1)+i}-\left(cb\alpha\beta+ca\beta\right)}, n\geq 1, \quad (3.1.57)$$

şeklinde yazılabilir. Denklem (3.1.57)’ de $i\in\{0,1,2,3,4,5\}$ için $v_{6n+i}=v_n^{(i)}$ dönüşümü uygulanırsa

$$v_n^{(i)}=\frac{\left(ab\alpha\beta\gamma+cb\alpha\beta+ca\beta+ab\right)v_{n-1}^{(i)}-\left(cb\alpha\beta^2+ca\beta^2+ab\beta\right)}{\left(ab\alpha\gamma+cb\alpha+ac\right)v_{n-1}^{(i)}-\left(cb\alpha\beta+ca\beta\right)}, n\geq 1, \quad (3.1.58)$$

elde edilir. İşlemlerin daha kolay anlaşılabilmesi için $n\in\mathbb{N}_0$ için $v_n^{(i)}=\hat{v}_n$ dönüşümü tanımlansın. Bu durumda $v_0^{(i)}=\hat{v}_0$ olacağı açıktır. Buradan Denklem (3.1.58)

$$\hat{v}_{n+1}=\frac{\left(ab\alpha\beta\gamma+cb\alpha\beta+ca\beta+ab\right)\hat{v}_n-\left(cb\alpha\beta^2+ca\beta^2+ab\beta\right)}{\left(ab\alpha\gamma+cb\alpha+ac\right)\hat{v}_n-\left(cb\alpha\beta+ca\beta\right)}, n\in\mathbb{N}_0, \quad (3.1.59)$$

iyi bilinen Riccati fark denklemine indirgenir. Denklem (3.1.59)' de $n \in \mathbb{N}_0$ için $(ab\alpha\gamma + cb\alpha + ac)\hat{v}_n - (cb\alpha\beta + ca\beta) = \dfrac{q_{n+1}}{q_n}$ dönüşümü uygulanırsa

$$q_{n+2} = (ab\alpha\beta\gamma + ab)q_{n+1} - (a^2b^2\alpha\beta\gamma)q_n, \ n \in \mathbb{N}_0, \tag{3.1.60}$$

2. mertebeden sabit katsayılı lineer fark denklemine indirgenir. Denklem (3.1.60)'a ait karakteristik denklem $\lambda^2 - (ab\alpha\beta\gamma + ab)\lambda + a^2b^2\alpha\beta\gamma = 0$ şeklindedir. Karakteristik denklemin kökleri farklı ise, yani $(ab\alpha\beta\gamma + ab)^2 - 4(a^2b^2\alpha\beta\gamma) \neq 0$, kökler $\lambda_3 = ab\alpha\beta\gamma$ ve $\lambda_4 = ab$ olur. Buradan $q_0$ ve $q_1$ Denklem (3.1.60)' ün başlangıç şartları olmak üzere verilen başlangıç değer probleminin çözümü

$$q_n = \frac{(q_1 - q_0\lambda_4)\lambda_3^{\,n}}{\lambda_3 - \lambda_4} - \frac{(q_1 - q_0\lambda_3)\lambda_4^{\,n}}{\lambda_3 - \lambda_4}, n \in \mathbb{N}_0, \tag{3.1.61}$$

elde edilir. Öte yandan $(ab\alpha\gamma + cb\alpha + ac)\hat{v}_n - (cb\alpha\beta + ca\beta) = \dfrac{q_{n+1}}{q_n}$ dönüşümünden

$$\begin{aligned}(ab\alpha\gamma + cb\alpha + ac)\hat{v}_n - (cb\alpha\beta + ca\beta) &= \frac{(q_1 - q_0\lambda_4)\lambda_3^{\,n+1} - (q_1 - q_0\lambda_3)\lambda_4^{\,n+1}}{(q_1 - q_0\lambda_4)\lambda_3^{\,n} - (q_1 - q_0\lambda_3)\lambda_4^{\,n}} \\ &= \frac{\left(\dfrac{q_1}{q_0} - \lambda_4\right)\lambda_3^{\,n+1} - \left(\dfrac{q_1}{q_0} - \lambda_3\right)\lambda_4^{\,n+1}}{\left(\dfrac{q_1}{q_0} - \lambda_4\right)\lambda_3^{\,n} - \left(\dfrac{q_1}{q_0} - \lambda_3\right)\lambda_4^{\,n}}\end{aligned} \tag{3.1.62}$$

olur. Şimdi karmaşıklığı önlemek için $X_2 = ab\alpha\gamma + cb\alpha + ac, Y_2 = cb\alpha\beta + ca\beta$ ve $Z_2 = a^2b^2\alpha\beta\gamma$ olmak üzere Denklem (3.1.62)' den

$$\hat{v}_n = \frac{1}{X_2}\frac{(X_2\hat{v}_0 - Y_2)\left(\dfrac{\lambda_3^{\,n+1} - \lambda_4^{\,n+1}}{\lambda_3 - \lambda_4}\right) - \lambda_3\lambda_4\left(\dfrac{\lambda_3^{\,n} - \lambda_4^{\,n}}{\lambda_3 - \lambda_4}\right)}{(X_2\hat{v}_0 - Y_2)\left(\dfrac{\lambda_3^{\,n} - \lambda_4^{\,n}}{\lambda_3 - \lambda_4}\right) - \lambda_3\lambda_4\left(\dfrac{\lambda_3^{\,n-1} - \lambda_4^{\,n-1}}{\lambda_3 - \lambda_4}\right)} + \frac{Y_2}{X_2}, n \in \mathbb{N}_0, \tag{3.1.63}$$

elde edilir. $v_{6n+i} = v_n^{(i)} = \hat{v}_n$ değişken değişimleri ve Tanım 2.2.10. 'dan

$$v_{6n+i}=\frac{1}{X_2}\frac{\left(X_2v_i-Y_2\right)\hat{s}_{n+1}-Z_2\hat{s}_n}{\left(X_2v_i-Y_2\right)\hat{s}_n-Z_2\hat{s}_{n-1}}+\frac{Y_2}{X_2},n\in\mathbb{N}_0, \tag{3.1.64}$$

Yazılır. Burada $i\in\{0,1,2,3,4,5\}$ 'dir. Diğer taraftan $\left(ab\alpha\beta\gamma+ab\right)^2-4\left(a^2b^2\alpha\beta\gamma\right)=0$ ise, yani karakteristik denklemin kökleri çakışık ise $\lambda_3=\lambda_4=\dfrac{ab\alpha\beta\gamma+ab}{2}$ olup Denklem (3.1.60) 'ün çözümü

$$q_n=c_1\lambda_3{}^n+c_2n\lambda_3{}^n,\ n\in\mathbb{N}_0, \tag{3.1.65}$$

şeklinde olur. $q_0$ ve $q_1$ Denklem (3.1.65)'in başlangıç şartları olmak üzere bu koşullar altındaki başlangıç değer probleminin çözümü

$$q_n=q_0\lambda_3{}^n+\left(q_1-q_0\lambda_3\right)n\lambda_3{}^{n-1},\ n\in\mathbb{N}_0, \tag{3.1.66}$$

elde edilir. Benzer şekilde $\left(ab\alpha\gamma+cb\alpha+ac\right)\hat{v}_n-\left(cb\alpha\beta+ca\beta\right)=\dfrac{q_{n+1}}{q_n}$ dönüşümünden

$$\left(ab\alpha\gamma+cb\alpha+ac\right)\hat{v}_n-\left(cb\alpha\beta+ca\beta\right)=\frac{q_0\lambda_3{}^{n+1}+\left(q_1-q_0\lambda_3\right)\left(n+1\right)\lambda_3{}^n}{q_0\lambda_3{}^n+\left(q_1-q_0\lambda_3\right)n\lambda_3{}^{n-1}} \tag{3.1.67}$$

bulunur. Şimdi $X_2=ab\alpha\gamma+cb\alpha+ac,\ Y_2=cb\alpha\beta+ca\beta$ dönüşümleri dikkate alınırsa Denklem (3.1.67)' tan

$$\hat{v}_n=\frac{1}{X_2}\frac{\left(X_2\hat{v}_0-Y_2\right)\left(n+1\right)\lambda_3{}^n-n\lambda_3{}^{n+1}}{\left(X_2\hat{v}_0-Y_2\right)n\lambda_3{}^{n-1}-\left(n-1\right)\lambda_3{}^n}+\frac{Y_2}{X_2},\ n\in\mathbb{N}_0, \tag{3.1.68}$$

elde edilir. $v_{6n+i}=v_n^{(i)}=\hat{v}_n$ değişken değişimlerinden

$$v_{6n+i}=\frac{1}{X_2}\frac{\left(X_2v_i-Y_2\right)\left(n+1\right)\lambda_3{}^n-n\lambda_3{}^{n+1}}{\left(X_2v_i-Y_2\right)n\lambda_3{}^{n-1}-\left(n-1\right)\lambda_3{}^n}+\frac{Y_2}{X_2},n\in\mathbb{N}_0, \tag{3.1.69}$$

olarak bulunur. Burada $i\in\{0,1,2,3,4,5\}$ 'dir.

Sistem (3.1.41)'de ikinci denklem, üçüncü denklemde yerine yazılırsa ve elde edilen ifadede birinci denklem yerine yazılırsa

$$w_{n+1}=\left(\gamma+\frac{c}{a}\right)-\frac{c\beta}{a\left[\left(\beta+\frac{b}{c}\right)-\frac{b\alpha}{c\left(\left(\alpha+\frac{a}{b}\right)-\frac{a\gamma}{bw_{n-5}}\right)}\right]}$$

$$=\left(\frac{a\gamma+c}{a}\right)-\frac{c\beta}{a\left[\frac{\left(bc\alpha\beta+ac\beta+ab\right)w_{n-5}-ab\gamma-ac\beta\gamma}{c\left(\left(b\alpha+a\right)w_{n-5}-a\gamma\right)}\right]}$$

$$=\left(\frac{\left(a\gamma+c\right)\left(\left(bc\alpha\beta+ac\beta+ab\right)w_{n-5}-ab\gamma-ac\beta\gamma\right)-\left(\left(c^2b\alpha\beta+ac^2\beta\right)w_{n-5}-ac^2\beta\gamma\right)}{a\left(\left(bc\alpha\beta+ac\beta+ab\right)w_{n-5}-ab\gamma-ac\beta\gamma\right)}\right) \quad (3.1.70)$$

$$=\frac{a\left(\left(bc\alpha\beta\gamma+ac\beta\gamma+ab\gamma+bc\right)w_{n-5}-\left(ab\gamma^2+ac\beta\gamma^2+bc\gamma\right)\right)}{a\left(\left(bc\alpha\beta+ac\beta+ab\right)w_{n-5}-\left(ab\gamma+ac\beta\gamma\right)\right)}$$

$$=\frac{\left(bc\alpha\beta\gamma+ac\beta\gamma+ab\gamma+bc\right)w_{n-5}-\left(ab\gamma^2+ac\beta\gamma^2+bc\gamma\right)}{\left(bc\alpha\beta+ac\beta+ab\right)w_{n-5}-\left(ab\gamma+ac\beta\gamma\right)},n\geq 5,$$

Riccati fark denklemi haline gelir. Denklem (3.1.70) de n yerine $n\to 6n+i$ değişken değişimi yapılırsa (3.1.70) denkleminde $i\in\{0,1,2,3,4,5\}$ için

$$w_{6n+i}=\frac{\left(bc\alpha\beta\gamma+ac\beta\gamma+ab\gamma+bc\right)w_{6(n-1)+i}-\left(ab\gamma^2+ac\beta\gamma^2+bc\gamma\right)}{\left(bc\alpha\gamma+ac\beta+ab\right)w_{6(n-1)+i}-\left(ab\gamma+ac\beta\gamma\right)},\ n\geq 1, \quad (3.1.71)$$

şeklinde yazılabilir. Denklem (3.1.71)' de $i\in\{0,1,2,3,4,5\}$ için $w_{6n+i}=w_n^{(i)}$ dönüşümü uygulanırsa

$$w_n^{(i)}=\frac{\left(bc\alpha\beta\gamma+ac\beta\gamma+ab\gamma+bc\right)w_{n-1}^{(i)}-\left(ab\gamma^2+ac\beta\gamma^2+bc\gamma\right)}{\left(bc\alpha\beta+ac\beta+ab\right)w_{n-1}^{(i)}-\left(ab\gamma+ac\beta\gamma\right)},n\geq 1, \quad (3.1.72)$$

elde edilir. İşlemlerin daha kolay anlaşılabilmesi için $n\in\mathbb{N}_0$ için $w_n^{(i)}=\hat{w}_n$ dönüşümü tanımlansın. Bu durumda $w_0^{(i)}=\hat{w}_0$ olacağı açıktır. Buradan Denklem (3.1.72)

$$\hat{w}_{n+1}=\frac{\left(bc\alpha\beta\gamma+ac\beta\gamma+ab\gamma+bc\right)\hat{w}_n-\left(ab\gamma^2+ac\beta\gamma^2+bc\gamma\right)}{\left(bc\alpha\beta+ac\beta+ab\right)\hat{w}_n-\left(ab\gamma+ac\beta\gamma\right)},n\in\mathbb{N}_0, \quad (3.1.73)$$

iyi bilinen Riccati fark denklemine indirgenir. Denklem (3.1.73)' de $n \in \mathbb{N}_0$ için $(bc\alpha\beta + ac\beta + ab)\hat{w}_n - (ab\gamma + ac\beta\gamma) = \frac{r_{n+1}}{r_n}$ dönüşümü uygulanırsa

$$r_{n+2} = (bc\alpha\beta\gamma + bc)r_{n+1} - (b^2c^2\alpha\beta\gamma)r_n, \ n \in \mathbb{N}_0, \tag{3.1.74}$$

2. mertebeden sabit katsayılı lineer fark denklemine indirgenir. Denklem (3.1.74)'e ait karakteristik denklem $\lambda^2 - (bc\alpha\beta\gamma + bc)\lambda + (b^2c^2\alpha\beta\gamma) = 0$ şeklindedir. Karakteristik denklemin kökleri farklı ise, yani $(bc\alpha\beta\gamma + bc)^2 - 4(b^2c^2\alpha\beta\gamma) \neq 0$, kökler $\lambda_5 = bc\alpha\beta\gamma$ ve $\lambda_6 = bc$ olur. Buradan $r_0$ ve $r_1$ Denklem (3.1.74)'ün başlangıç şartları olmak üzere verilen başlangıç değer probleminin çözümü

$$r_n = \frac{(r_1 - r_0\lambda_6)\lambda_5^n}{\lambda_5 - \lambda_6} - \frac{(r_1 - r_0\lambda_5)\lambda_6^n}{\lambda_5 - \lambda_6}, \ n \in \mathbb{N}_0, \tag{3.1.75}$$

elde edilir. Öte yandan $(bc\alpha\beta + ac\beta + ab)\hat{w}_n - (ab\gamma + ac\beta\gamma) = \frac{r_{n+1}}{r_n}$ dönüşümünden

$$\begin{aligned}(bc\alpha\beta + ac\beta + ab)\hat{w}_n - (ab\gamma + ac\beta\gamma) &= \frac{(r_1 - r_0\lambda_6)\lambda_5^{n+1} - (r_1 - r_0\lambda_5)\lambda_6^{n+1}}{(r_1 - r_0\lambda_6)\lambda_5^n - (r_1 - r_0\lambda_5)\lambda_6^n} \\ &= \frac{\left(\frac{r_1}{r_0} - \lambda_6\right)\lambda_5^{n+1} - \left(\frac{r_1}{r_0} - \lambda_5\right)\lambda_6^{n+1}}{\left(\frac{r_1}{r_0} - \lambda_6\right)\lambda_5^n - \left(\frac{r_1}{r_0} - \lambda_5\right)\lambda_6^n}\end{aligned} \tag{3.1.76}$$

olur. Şimdi karmaşıklığı önlemek için $X_3 = bc\alpha\beta + ac\beta + ab, Y_3 = ab\gamma + ac\beta\gamma$ ve $Z_3 = b^2c^2\alpha\beta\gamma$ olmak üzere Denklem (3.1.76)' den

$$\hat{w}_n = \frac{1}{X_3}\frac{(X_3\hat{w}_0 - Y_3)\left(\frac{\lambda_5^{n+1} - \lambda_6^{n+1}}{\lambda_5 - \lambda_6}\right) - \lambda_5\lambda_6\left(\frac{\lambda_5^n - \lambda_6^n}{\lambda_5 - \lambda_6}\right)}{(X_3\hat{w}_0 - Y_3)\left(\frac{\lambda_5^n - \lambda_6^n}{\lambda_5 - \lambda_6}\right) - \lambda_5\lambda_6\left(\frac{\lambda_5^{n-1} - \lambda_6^{n-1}}{\lambda_5 - \lambda_6}\right)} + \frac{Y_3}{X_3}, n \in \mathbb{N}_0, \tag{3.1.77}$$

elde edilir. $w_{6n+i} = w_n^{(i)} = \hat{w}_n$ değişken değişimleri ve Tanım 2.2.10. 'dan

$$w_{6n+i}=\frac{1}{X_3}\frac{\left(X_3w_i-Y_3\right)\tilde{s}_{n+1}-Z_3\tilde{s}_n}{\left(X_3w_i-Y_3\right)\tilde{s}_n-Z_3\tilde{s}_{n-1}}+\frac{Y_3}{X_3},n\in\mathbb{N}_0, \tag{3.1.78}$$

bulunur. Burada $i\in\{0,1,2,3,4,5\}$'dir. Diğer taraftan $\left(bc\alpha\beta\gamma+bc\right)^2-4\left(b^2c^2\alpha\beta\gamma\right)=0$ ise, yani karakteristik denklemin kökleri çakışık ise $\lambda_5=\lambda_6=\frac{bc\alpha\beta\gamma+bc}{2}$ olup Denklem (3.1.74)' ün çözümü

$$r_n=c_1\lambda_5{}^n+c_2n\lambda_5{}^n,\ n\in\mathbb{N}_0, \tag{3.1.79}$$

şeklinde olur. $r_0$ ve $r_1$ Denklem (3.1.79)'in başlangıç şartları olmak üzere bu koşullar altındaki başlangıç değer probleminin çözümü

$$r_n=r_0\lambda_5{}^n+\left(r_1-r_0\lambda_5\right)n\lambda_5{}^{n-1},\ n\in\mathbb{N}_0, \tag{3.1.80}$$

elde edilir. Benzer şekilde $\left(bc\alpha\beta+ac\beta+ab\right)\hat{w}_n-\left(ab\gamma+ac\beta\gamma\right)=\frac{r_{n+1}}{r_n}$ dönüşümünden

$$\left(bc\alpha\beta+ac\beta+ab\right)\hat{w}_n-\left(ab\gamma+ac\beta\gamma\right)=\frac{r_0\lambda_5{}^{n+1}+\left(r_1-r_0\lambda_5\right)\left(n+1\right)\lambda_5{}^n}{r_0\lambda_5{}^n+\left(r_1-r_0\lambda_5\right)n\lambda_5{}^{n-1}} \tag{3.1.81}$$

bulunur. Şimdi $X_3=bc\alpha\beta+ac\beta+ab$ ve $Y_3=ab\gamma+ac\beta\gamma$ ifadeleri dikkate alınırsa Denklem (3.1.81)' den

$$\hat{w}_n=\frac{1}{X_3}\frac{\left(X_3\hat{w}_0-Y_3\right)\left(n+1\right)\lambda_5{}^n-n\lambda_5{}^{n+1}}{\left(X_3\hat{w}_0-Y_3\right)n\lambda_5{}^{n-1}-\left(n-1\right)\lambda_5{}^n}+\frac{Y_3}{X_3},n\in\mathbb{N}_0, \tag{3.1.82}$$

elde edilir. $w_{6n+i}=w_n^{(i)}=\hat{w}_n$ değişken değişimlerinden

$$w_{6n+i}=\frac{1}{X_3}\frac{\left(X_3w_i-Y_3\right)\left(n+1\right)\lambda_5{}^n-n\lambda_5{}^{n+1}}{\left(X_3w_i-Y_3\right)n\lambda_5{}^{n-1}-\left(n-1\right)\lambda_5{}^n}+\frac{Y_3}{X_3},n\in\mathbb{N}_0 \tag{3.1.83}$$

olarak bulunur. Burada $i\in\{0,1,2,3,4,5\}$'dir.

$\frac{x_n}{y_{n-1}}=u_n,\ \frac{y_n}{z_{n-1}}=v_n,\ \frac{z_n}{x_{n-1}}=w_n,\ n\in\mathbb{N}_0,$ dönüşümünden

$$x_n = u_n y_{n-1} = u_n v_{n-1} z_{n-2} = u_n v_{n-1} w_{n-2} x_{n-3},\ n \geq 2,$$
$$y_n = v_n z_{n-1} = v_n w_{n-1} x_{n-2} = v_n w_{n-1} u_{n-2} y_{n-3},\ n \geq 2, \quad (3.1.84)$$
$$z_n = w_n x_{n-1} = w_n u_{n-1} y_{n-2} = w_n u_{n-1} v_{n-2} z_{n-3},\ n \geq 2,$$

bulunur. Buradan

$$x_{6m+i} = x_i \prod_{j=1}^{m} u_{6j+i} v_{6j+i-1} w_{6j+i-2} u_{6j+i-3} v_{6j+i-4} w_{6j+i-5},$$
$$y_{6m+i} = y_i \prod_{j=1}^{m} v_{6j+i} w_{6j+i-1} u_{6j+i-2} v_{6j+i-3} w_{6j+i-4} u_{6j+i-5}, \quad (3.1.85)$$
$$z_{6m+i} = z_i \prod_{j=1}^{m} w_{6j+i} u_{6j+i-1} v_{6j+i-2} w_{6j+i-3} u_{6j+i-4} v_{6j+i-5},$$

elde edilir. Burada $m \in \mathbb{N}_0,\ i \in \{-1, 0, 1, 2, 3, 4\}$ 'dir.

Şimdi Sistem (3.1.34)' ün açık formda çözümlerini vermeden önce karışıklığı önlemek için tezin bundan sonraki kısmı için kullanılacak olan aşağıdaki notasyonlar verilecektir.

$$\begin{cases} X_1 = ac\beta\gamma + ab\gamma + bc, \\ Y_1 = ab\alpha\gamma + bc\alpha, \\ Z_1 = a^2c^2\alpha\beta\gamma, \\ X_2 = ab\alpha\gamma + cb\alpha + ac, \\ Y_2 = cb\alpha\beta + ca\beta, \\ Z_2 = a^2b^2\alpha\beta\gamma, \\ X_3 = bc\alpha\beta + ac\beta + ab, \\ Y_3 = ab\gamma + ac\beta\gamma, \\ Z_3 = b^2c^2\alpha\beta\gamma. \end{cases}$$

Sistem (3.1.34)' ün açık formda çözümleri $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ ve $\lambda_6$ 'nın birbirine eşit ve farklı olmalarına göre 8 durum söz konusudur.

**1. durum** $\lambda_1 \neq \lambda_2,\ \lambda_3 \neq \lambda_4,\ \lambda_5 \neq \lambda_6$ **ise**

Denklem (3.1.85)'de $i = -1$ ve $m \in \mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$x_{6m-1} = x_{-1}\prod_{j=1}^{m} u_{6j-1}v_{6j-2}w_{6j-3}u_{6j-4}v_{6j-5}w_{6j-6}$$

$$\begin{aligned} &= x_{-1}\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{(X_1u_5-Y_1)s_j-Z_1s_{j-1}}{(X_1u_5-Y_1)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_4-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}{(X_2v_4-Y_2)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right] \\ &\times\left[\frac{1}{X_3}\frac{(X_3w_3-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{(X_3w_3-Y_3)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_2-Y_1)s_j-Z_1s_{j-1}}{(X_1u_2-Y_1)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right] \\ &\times\left[\frac{1}{X_2}\frac{(X_2v_1-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}{(X_2v_1-Y_2)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_0-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{(X_3w_0-Y_3)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right], \end{aligned} \tag{3.1.86}$$

$$y_{6m-1} = y_{-1}\prod_{j=1}^{m} v_{6j-1}w_{6j-2}u_{6j-3}v_{6j-4}w_{6j-5}u_{6j-6}$$

$$\begin{aligned} &= y_{-1}\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{(X_2v_5-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}{(X_2v_5-Y_2)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_4-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{(X_3w_4-Y_3)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right] \\ &\times\left[\frac{1}{X_1}\frac{(X_1u_3-Y_1)s_j-Z_1s_{j-1}}{(X_1u_3-Y_1)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_2-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}{(X_2v_2-Y_2)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right] \\ &\times\left[\frac{1}{X_3}\frac{(X_3w_1-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{(X_3w_1-Y_3)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_0-Y_1)s_j-Z_1s_{j-1}}{(X_1u_0-Y_1)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right], \end{aligned} \tag{3.1.87}$$

$$z_{6m-1} = z_{-1}\prod_{j=1}^{m} w_{6j-1}u_{6j-2}v_{6j-3}w_{6j-4}u_{6j-5}v_{6j-6}$$

$$\begin{aligned} &= z_{-1}\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{(X_3w_5-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{(X_3w_5-Y_3)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_4-Y_1)s_j-Z_1s_{j-1}}{(X_1u_4-Y_1)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right] \\ &\times\left[\frac{1}{X_2}\frac{(X_2v_3-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}{(X_2v_3-Y_2)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_2-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{(X_3w_2-Y_3)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right] \\ &\times\left[\frac{1}{X_1}\frac{(X_1u_1-Y_1)s_j-Z_1s_{j-1}}{(X_1u_1-Y_1)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_0-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}{(X_2v_0-Y_2)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right] \end{aligned} \tag{3.1.88}$$

olur.

Denklem (3.1.85)'de $i=0$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$x_{6m}=x_0\prod_{j=1}^{m}u_{6j}v_{6j-1}w_{6j-2}u_{6j-3}v_{6j-4}w_{6j-5}$$

$$\begin{aligned}&=x_0\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{\left(X_1u_0-Y_1\right)s_{j+1}-Z_1s_j}{\left(X_1u_0-Y_1\right)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_5-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}{\left(X_2v_5-Y_2\right)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\\&\times\left[\frac{1}{X_3}\frac{\left(X_3w_4-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{\left(X_3w_4-Y_3\right)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_3-Y_1\right)s_j-Z_1s_{j-1}}{\left(X_1u_3-Y_1\right)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right]\\&\times\left[\frac{1}{X_2}\frac{\left(X_2v_2-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}{\left(X_2v_2-Y_2\right)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_1-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{\left(X_3w_1-Y_3\right)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right],\end{aligned}\tag{3.1.89}$$

$$y_{6m}=y_0\prod_{j=1}^{m}v_{6j}w_{6j-1}u_{6j-2}v_{6j-3}w_{6j-4}u_{6j-5}$$

$$\begin{aligned}&=y_0\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{\left(X_2v_0-Y_2\right)\hat{s}_{j+1}-Z_2\hat{s}_j}{\left(X_2v_0-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_5-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{\left(X_3w_5-Y_3\right)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right]\\&\times\left[\frac{1}{X_1}\frac{\left(X_1u_4-Y_1\right)s_j-Z_1s_{j-1}}{\left(X_1u_4-Y_1\right)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_3-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}{\left(X_2v_3-Y_2\right)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\\&\times\left[\frac{1}{X_3}\frac{\left(X_3w_2-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{X_3w_2-Y_3\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_1-Y_1\right)s_j-Z_1s_{j-1}}{\left(X_1u_1-Y_1\right)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right],\end{aligned}\tag{3.1.90}$$

$$z_{6m}=z_0\prod_{j=1}^{m}w_{6j}u_{6j-1}v_{6j-2}w_{6j-3}u_{6j-4}v_{6j-5}$$

$$\begin{aligned}&=z_0\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{\left(X_3w_0Y_3\right)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{\left(X_3w_0Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_5-Y_1\right)s_j-Z_1s_{j-1}}{\left(X_1u_5-Y_1\right)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right]\\&\times\left[\frac{1}{X_2}\frac{\left(X_2v_4-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}{\left(X_2v_4-Y_2\right)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_3-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{\left(X_3w_3-Y_3\right)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right]\\&\times\left[\frac{1}{X_1}\frac{\left(X_1u_2-Y_1\right)s_j-Z_1s_{j-1}}{\left(X_1u_2-Y_1\right)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_4-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}{\left(X_2v_4-Y_2\right)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\end{aligned}\tag{3.1.91}$$

elde edilir. Denklem (3.1.85)'de $i=1$ ve $m\in\mathbb{N}_0$ için $x_{6m+i},y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$
\begin{aligned}
x_{6m+1} &= x_1\prod_{j=1}^{m} u_{6j+1}v_{6j}w_{6j-1}u_{6j-2}v_{6j-3}w_{6j-4} \\
&= x_1\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{\left(X_1u_1-Y_1\right)s_{j+1}-Z_1s_j}{\left(X_1u_1-Y_1\right)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_0-Y_2\right)\hat{s}_{j+1}-Z_2\hat{s}_j}{\left(X_2v_0-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right] \\
&\times\left[\frac{1}{X_3}\frac{\left(X_3w_5-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{\left(X_3w_5-Y_3\right)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_4-Y_1\right)s_j-Z_1s_{j-1}}{\left(X_1u_4-Y_1\right)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right] \\
&\times\left[\frac{1}{X_2}\frac{\left(X_2v_3-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}{\left(X_2v_3-Y_2\right)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_2-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{\left(X_3w_2-Y_3\right)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right],
\end{aligned}
\tag{3.1.92}
$$

$$
\begin{aligned}
y_{6m+1} &= y_1\prod_{j=1}^{m} v_{6j+1}w_{6j}u_{6j-1}v_{6j-2}w_{6j-3}u_{6j-4} \\
&= y_1\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{\left(X_2v_1-Y_2\right)\hat{s}_{j+1}-Z_2\hat{s}_j}{\left(X_2v_1-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_0-Y_3\right)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{\left(X_3w_0-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right] \\
&\times\left[\frac{1}{X_1}\frac{\left(X_1u_5-Y_1\right)s_j-Z_1s_{j-1}}{\left(X_1u_5-Y_1\right)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_4-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}{\left(X_2v_4-Y_2\right)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right] \\
&\times\left[\frac{1}{X_3}\frac{\left(X_3w_3-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{\left(X_3w_3-Y_3\right)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_2-Y_1\right)s_j-Z_1s_{j-1}}{\left(X_1u_2-Y_1\right)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right],
\end{aligned}
\tag{3.1.93}
$$

$$
\begin{aligned}
z_{6m+1} &= z_1\prod_{j=1}^{m} w_{6j+1}u_{6j}v_{6j-1}w_{6j-2}u_{6j-3}v_{6j-4} \\
&= z_1\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{\left(X_3w_1-Y_3\right)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{\left(X_3w_1-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_0-Y_1\right)s_{j+1}-Z_1s_j}{\left(X_1u_0-Y_1\right)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right] \\
&\times\left[\frac{1}{X_2}\frac{\left(X_2v_5-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}{\left(X_2v_5-Y_2\right)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_4-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{\left(X_3w_4-Y_3\right)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right] \\
&\times\left[\frac{1}{X_1}\frac{\left(X_1u_3-Y_1\right)s_j-Z_1s_{j-1}}{\left(X_1u_3-Y_1\right)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_2-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}{\left(X_2v_2-Y_2\right)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]
\end{aligned}
\tag{3.1.94}
$$

olur. Denklem (3.1.85)'de $i=2$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$x_{6m+2} = x_2\prod_{j=1}^{m}u_{6j+2}v_{6j+1}w_{6j}u_{6j-1}v_{6j-2}w_{6j-3}$$

$$= x_2\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{(X_1u_2-Y_1)s_{j+1}-Z_1s_j}{(X_1u_2-Y_1)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_1-Y_2)\hat{s}_{j+1}-Z_2\hat{s}_j}{(X_2v_1-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]$$
$$\times\left[\frac{1}{X_3}\frac{(X_3w_0-Y_3)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{(X_3w_0-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_5-Y_1)s_j-Z_1s_{j-1}}{(X_1u_5-Y_1)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right] \qquad (3.1.95)$$
$$\times\left[\frac{1}{X_2}\frac{(X_2v_4-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}{(X_2v_4-Y_2)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_3-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{(X_3w_3-Y_3)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right],$$

$$y_{6m+2} = y_2\prod_{j=1}^{m}v_{6j+2}w_{6j+1}u_{6j}v_{6j-1}w_{6j-2}u_{6j-3}$$

$$= y_2\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{(X_2v_2-Y_2)\hat{s}_{j+1}-Z_2\hat{s}_j}{(X_2v_2-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_1-Y_3)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{(X_3w_1-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]$$
$$\times\left[\frac{1}{X_1}\frac{(X_1u_0-Y_1)s_{j+1}-Z_1s_j}{(X_1u_0-Y_1)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_5-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}{(X_2v_5-Y_2)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right] \qquad (3.1.96)$$
$$\times\left[\frac{1}{X_3}\frac{(X_3w_4-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{(X_3w_4-Y_3)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_3-Y_1)s_j-Z_1s_{j-1}}{(X_1u_3-Y_1)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right],$$

$$z_{6m+2} = z_2\prod_{j=1}^{m}w_{6j+2}u_{6j+1}v_{6j}w_{6j-1}u_{6j-2}v_{6j-3}$$

$$= z_2\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{(X_3w_2-Y_3)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{(X_3w_2-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_1-Y_1)s_{j+1}-Z_1s_j}{(X_1u_1-Y_1)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]$$
$$\times\left[\frac{1}{X_2}\frac{(X_2v_0-Y_2)\hat{s}_{j+1}-Z_2\hat{s}_j}{(X_2v_0-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_5-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{(X_3w_5-Y_3)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right] \qquad (3.1.97)$$
$$\times\left[\frac{1}{X_1}\frac{(X_1u_4-Y_1)s_j-Z_1s_{j-1}}{(X_1u_4-Y_1)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_3-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}{(X_2v_3-Y_2)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]$$

olur. Denklem (3.1.85)'de $i=3$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$
\begin{aligned}
x_{6m+3} &= x_3\prod_{j=1}^{m}u_{6j+3}v_{6j+2}w_{6j+1}u_{6j}v_{6j-1}w_{6j-2}\\
&= x_3\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{\left(X_1u_3-Y_1\right)s_{j+1}-Z_1s_j}{\left(X_1u_3-Y_1\right)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_2-Y_2\right)\hat{s}_{j+1}-Z_2\hat{s}_j}{\left(X_2v_2-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]\\
&\times\left[\frac{1}{X_3}\frac{\left(X_3w_1-Y_3\right)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{\left(X_3w_1-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_0-Y_1\right)s_{j+1}-Z_1s_j}{\left(X_1u_0-Y_1\right)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]\\
&\times\left[\frac{1}{X_2}\frac{\left(X_2v_5-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}{\left(X_2v_5-Y_2\right)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_4-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{\left(X_3w_4-Y_3\right)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right],
\end{aligned}
\tag{3.1.98}
$$

$$
\begin{aligned}
y_{6m+3} &= y_3\prod_{j=1}^{m}v_{6j+3}w_{6j+2}u_{6j+1}v_{6j}w_{6j-1}u_{6j-2}\\
&= y_3\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{\left(X_2v_3-Y_2\right)\hat{s}_{j+1}-Z_2\hat{s}_j}{\left(X_2v_3-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_2-Y_3\right)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{\left(X_3w_2-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]\\
&\times\left[\frac{1}{X_1}\frac{\left(X_1u_1-Y_1\right)s_{j+1}-Z_1s_j}{\left(X_1u_1-Y_1\right)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_0-Y_2\right)\hat{s}_{j+1}-Z_2\hat{s}_j}{\left(X_2v_0-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]\\
&\times\left[\frac{1}{X_3}\frac{\left(X_3w_5-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{\left(X_3w_5-Y_3\right)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_4-Y_1\right)s_j-Z_1s_{j-1}}{\left(X_1u_4-Y_1\right)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right],
\end{aligned}
\tag{3.1.99}
$$

$$
\begin{aligned}
z_{6m+3} &= z_3\prod_{j=1}^{m}w_{6j+3}u_{6j+2}v_{6j+1}w_{6j}u_{6j-1}v_{6j-2}\\
&= z_3\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{\left(X_3w_3-Y_3\right)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{\left(X_3w_3-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_2-Y_1\right)s_{j+1}-Z_1s_j}{\left(X_1u_2-Y_1\right)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]\\
&\times\left[\frac{1}{X_2}\frac{\left(X_2v_1-Y_2\right)\hat{s}_{j+1}-Z_2\hat{s}_j}{\left(X_2v_1-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_0-Y_3\right)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{\left(X_3w_0-Y_3\right)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]\\
&\times\left[\frac{1}{X_1}\frac{\left(X_1u_5-Y_1\right)s_j-Z_1s_{j-1}}{\left(X_1u_5-Y_1\right)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_4-Y_2\right)\hat{s}_j-Z_2\hat{s}_{j-1}}{\left(X_2v_4-Y_2\right)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right]
\end{aligned}
\tag{3.1.100}
$$

olur. Denklem (3.1.85)’de $i=4$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$\begin{aligned} x_{6m+4} &= x_4\prod_{j=1}^{m} u_{6j+4}v_{6j+3}w_{6j+2}u_{6j+1}v_{6j}w_{6j-1} \\ &= x_4\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{(X_1u_4-Y_1)s_{j+1}-Z_1s_j}{(X_1u_4-Y_1)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_3-Y_2)\hat{s}_{j+1}Z_2\hat{s}_j}{(X_2v_3-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right] \\ &\times\left[\frac{1}{X_3}\frac{(X_3w_2-Y_3)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{(X_3w_2-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_1-Y_1)s_{j+1}-Z_1s_j}{(X_1u_1-Y_1)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right] \\ &\times\left[\frac{1}{X_2}\frac{(X_2v_0-Y_2)\hat{s}_{j+1}-Z_2\hat{s}_j}{(X_2v_0-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_5-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}{(X_3w_5-Y_3)\tilde{s}_{j-1}-Z_3\tilde{s}_{j-2}}+\frac{Y_3}{X_3}\right], \end{aligned} \tag{3.1.101}$$

$$\begin{aligned} y_{6m+4} &= y_4\prod_{j=1}^{m} v_{6j+4}w_{6j+3}u_{6j+2}v_{6j+1}w_{6j}u_{6j-1} \\ &= y_4\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{(X_2v_4-Y_2)\hat{s}_{j+1}-Z_2\hat{s}_j}{(X_2v_4-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_3-Y_3)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{(X_3w_3-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right] \\ &\times\left[\frac{1}{X_1}\frac{(X_1u_2-Y_1)s_{j+1}-Z_1s_j}{(X_1u_2-Y_1)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_1-Y_2)\hat{s}_{j+1}-Z_2\hat{s}_j}{(X_2v_1-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right] \\ &\times\left[\frac{1}{X_3}\frac{(X_3w_0-Y_3)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{(X_3w_0-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_5-Y_1)s_j-Z_1s_{j-1}}{(X_1u_5-Y_1)s_{j-1}-Z_1s_{j-2}}+\frac{Y_1}{X_1}\right], \end{aligned} \tag{3.1.102}$$

$$\begin{aligned} z_{6m+4} &= z_4\prod_{j=1}^{m} w_{6j+4}u_{6j+3}v_{6j+2}w_{6j+1}u_{6j}v_{6j-1} \\ &= z_4\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{(X_3w_4-Y_3)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{(X_3w_4-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_3-Y_1)s_{j+1}-Z_1s_j}{(X_1u_3-Y_1)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right] \\ &\times\left[\frac{1}{X_2}\frac{(X_2v_2-Y_2)\hat{s}_{j+1}-Z_2\gamma\hat{s}_j}{(X_2v_2-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_1-Y_3)\tilde{s}_{j+1}-Z_3\tilde{s}_j}{(X_3w_1-Y_3)\tilde{s}_j-Z_3\tilde{s}_{j-1}}+\frac{Y_3}{X_3}\right] \\ &\times\left[\frac{1}{X_1}\frac{(X_1u_0-Y_1)s_{j+1}-Z_1s_j}{(X_1u_0-Y_1)s_j-Z_1s_{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_5-Y_2)\hat{s}_j-Z_2\hat{s}_{j-1}}{(X_2v_5-Y_2)\hat{s}_{j-1}-Z_2\hat{s}_{j-2}}+\frac{Y_2}{X_2}\right] \end{aligned} \tag{3.1.103}$$

olur. $\lambda_1\neq\lambda_2, \lambda_3\neq\lambda_4\ ve\ \lambda_5\neq\lambda_6$ durumu için (3.1.86)-(3.1.103) çözümleri dikkate alındığında Sistem (3.1.34)' ün çözümlerini daha kısa formda yazılacak olursa, bazı basit hesaplamalar sonrası Sistem (3.1.34)' ün, $m\in\mathbb{N}_0\ ve\ \ i\in\{-1,0,1,2,3,4\}$ için, çözümü

$$x_{6m+i}=x_i\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{\left(X_1u_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i}{6}\right\rfloor}}{\left(X_1u_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_2\hat{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}}{\left(X_2v_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right]$$

$$\times\left[\frac{1}{X_3}\frac{\left(X_3w_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_3w_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}}{\left(X_1u_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right] \quad (3.1.104)$$

$$\times\left[\frac{1}{X_2}\frac{\left(X_2v_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_2\hat{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}}{\left(X_2v_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_3w_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right],$$

$$y_{6m+i}=y_i\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{\left(X_2v_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i}{6}\right\rfloor}-Z_2\hat{s}_{j+\left\lfloor\frac{i}{6}\right\rfloor}}{\left(X_2v_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\left\lfloor\frac{i}{6}\right\rfloor}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}}{\left(X_3w_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right]$$

$$\times\left[\frac{1}{X_1}\frac{\left(X_1u_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_1u_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}Z_2\hat{s}_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}}{\left(X_2v_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right] \quad (3.1.105)$$

$$\times\left[\frac{1}{X_3}\frac{\left(X_3w_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}}{\left(X_3w_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_1u_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right],$$

$$z_{6m+i}=z_i\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{\left(X_3w_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i}{6}\right\rfloor}}{\left(X_3w_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{\left(X_1u_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}}{\left(X_1u_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right]$$

$$\times\left[\frac{1}{X_2}\frac{\left(X_2v_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_2\hat{s}_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_2v_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{\left(X_3w_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}}{\left(X_3w_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right] \quad (3.1.106)$$

$$\times\left[\frac{1}{X_1}\frac{\left(X_1u_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}}{\left(X_1u_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{\left(X_2v_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_2\hat{s}_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_2v_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right]$$

şeklinde olacaktır. Burada $\lfloor x \rfloor$ literatürde iyi bilinen tam değeri ifade eder.

**2. durum $\lambda_1=\lambda_2$, $\lambda_3=\lambda_4$, $\lambda_5=\lambda_6$ ise**

Denklem (3.1.85)'de $i=-1$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$
\begin{aligned}
x_{6m-1} &= x_{-1}\prod_{j=1}^{m}u_{6j-1}v_{6j-2}w_{6j-3}u_{6j-4}v_{6j-5}w_{6j-6}\\
&= x_{-1}\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{(X_1u_5-Y_1)j\lambda_1^{j-1}-(j-1)\lambda_1^{j}}{(X_1u_5-Y_1)(j-1)\lambda_1^{j-2}-(j-2)\lambda_1^{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_4-Y_2)j\lambda_3^{j-1}-(j-1)\lambda_3^{j}}{(X_2v_4-Y_2)(j-1)\lambda_3^{j-2}-(j-2)\lambda_3^{j-1}}+\frac{Y_2}{X_2}\right] \quad (3.1.107)\\
&\times\left[\frac{1}{X_3}\frac{(X_3\hat{w}_3-Y_3)j\lambda_5^{j-1}-(j-1)\lambda_5^{j}}{(X_3\hat{w}_3-Y_3)(j-1)\lambda_5^{j-2}-(j-2)\lambda_5^{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_2-Y_1)j\lambda_1^{j-1}-(j-1)\lambda_1^{j}}{(X_1u_2-Y_1)(j-1)\lambda_1^{j-2}-(j-2)\lambda_1^{j-1}}+\frac{Y_1}{X_1}\right]\\
&\times\left[\frac{1}{X_2}\frac{(X_2v_1-Y_2)j\lambda_3^{j-1}-(j-1)\lambda_3^{j}}{(X_2v_1-Y_2)(j-1)\lambda_3^{j-2}-(j-2)\lambda_3^{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3\hat{w}_0-Y_3)j\lambda_5^{j-1}-(j-1)\lambda_5^{j}}{(X_3\hat{w}_0-Y_3)(j-1)\lambda_5^{j-2}-(j-2)\lambda_5^{j-1}}+\frac{Y_3}{X_3}\right],
\end{aligned}
$$

$$
\begin{aligned}
y_{6m-1} &= y_{-1}\prod_{j=1}^{m}v_{6j-1}w_{6j-2}u_{6j-3}v_{6j-4}w_{6j-5}u_{6j-6}\\
&= y_{-1}\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{(X_2v_5-Y_2)j\lambda_3^{j-1}-(j-1)\lambda_3^{j}}{(X_2v_5-Y_2)(j-1)\lambda_3^{j-2}-(j-2)\lambda_3^{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3\hat{w}_4-Y_3)j\lambda_5^{j-1}-(j-1)\lambda_5^{j}}{(X_3\hat{w}_4-Y_3)(j-1)\lambda_5^{j-2}-(j-2)\lambda_5^{j-1}}+\frac{Y_3}{X_3}\right] \quad (3.1.108)\\
&\times\left[\frac{1}{X_1}\frac{(X_1u_3-Y_1)j\lambda_1^{j-1}-(j-1)\lambda_1^{j}}{(X_1u_3-Y_1)(j-1)\lambda_1^{j-2}-(j-2)\lambda_1^{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_2-Y_2)j\lambda_3^{j-1}-(j-1)\lambda_3^{j}}{(X_2v_2-Y_2)(j-1)\lambda_3^{j-2}-(j-2)\lambda_3^{j-1}}+\frac{Y_2}{X_2}\right]\\
&\times\left[\frac{1}{X_3}\frac{(X_3\hat{w}_1-Y_3)j\lambda_5^{j-1}-(j-1)\lambda_5^{j}}{(X_3\hat{w}_1-Y_3)(j-1)\lambda_5^{j-2}-(j-2)\lambda_5^{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_0-Y_1)j\lambda_1^{j-1}-(j-1)\lambda_1^{j}}{(X_1u_0-Y_1)(j-1)\lambda_1^{j-2}-(j-2)\lambda_1^{j-1}}+\frac{Y_1}{X_1}\right],
\end{aligned}
$$

$$
\begin{aligned}
z_{6m-1} &= z_{-1}\prod_{j=1}^{m}w_{6j-1}u_{6j-2}v_{6j-3}w_{6j-4}u_{6j-5}v_{6j-6}\\
&= z_{-1}\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{(X_3w_5-Y_3)j\lambda_5^{j-1}-(j-1)\lambda_5^{j}}{(X_3w_5-Y_3)(j-1)\lambda_5^{j-2}-(j-2)\lambda_5^{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_4-Y_1)j\lambda_1^{j-1}-(j-1)\lambda_1^{j}}{(X_1u_4-Y_1)(j-1)\lambda_1^{j-2}-(j-2)\lambda_1^{j-1}}+\frac{Y_1}{X_1}\right] \quad (3.1.109)\\
&\times\left[\frac{1}{X_2}\frac{(X_2v_3-Y_2)j\lambda_3^{j-1}-(j-1)\lambda_3^{j}}{(X_2v_3-Y_2)(j-1)\lambda_3^{j-2}-(j-2)\lambda_3^{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_2-Y_3)j\lambda_5^{j-1}-(j-1)\lambda_5^{j}}{(X_3w_2-Y_3)(j-1)\lambda_5^{j-2}-(j-2)\lambda_5^{j-1}}+\frac{Y_3}{X_3}\right]\\
&\times\left[\frac{1}{X_1}\frac{(X_1u_1-Y_1)j\lambda_1^{j-1}-(j-1)\lambda_1^{j}}{(X_1u_1-Y_1)(j-1)\lambda_1^{j-2}-(j-2)\lambda_1^{j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_0-Y_2)j\lambda_3^{j-1}-(j-1)\lambda_3^{j}}{(X_2v_0-Y_2)(j-1)\lambda_3^{j-2}-(j-2)\lambda_3^{j-1}}+\frac{Y_2}{X_2}\right]
\end{aligned}
$$

olur. Denklem (3.1.85)'de $i=0$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$
\begin{aligned}
x_{6m} &= x_{0}\prod_{j=1}^{m}u_{6j}v_{6j-1}w_{6j-2}u_{6j-3}v_{6j-4}w_{6j-5}\\
&= x_{0}\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{(X_1u_0-Y_1)(j+1)\lambda_1^{j}-j\lambda_1^{j+1}}{(X_1u_0-Y_1)j\lambda_1^{j-1}-(j-1)\lambda_1^{j}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_5-Y_2)j\lambda_3^{j-1}-(j-1)\lambda_3^{j}}{(X_2v_5-Y_2)(j-1)\lambda_3^{j-2}-(j-2)\lambda_3^{j-1}}+\frac{Y_2}{X_2}\right] \quad (3.1.110)\\
&\times\left[\frac{1}{X_3}\frac{(X_3w_4-Y_3)j\lambda_5^{j-1}-(j-1)\lambda_5^{j}}{(X_3w_4-Y_3)(j-1)\lambda_5^{j-2}-(j-2)\lambda_5^{j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_3-Y_1)j\lambda_1^{j-1}-(j-1)\lambda_1^{j}}{(X_1u_3-Y_1)(j-1)\lambda_1^{j-2}-(j-2)\lambda_1^{j-1}}+\frac{Y_1}{X_1}\right]\\
&\times\left[\frac{1}{X_2}\frac{(X_2v_2-Y_2)j\lambda_3^{j-1}-(j-1)\lambda_3^{j}}{(X_2v_2-Y_2)(j-1)\lambda_3^{j-2}-(j-2)\lambda_3^{j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_1-Y_3)j\lambda_5^{j-1}-(j-1)\lambda_5^{j}}{(X_3w_1-Y_3)(j-1)\lambda_5^{j-2}-(j-2)\lambda_5^{j-1}}+\frac{Y_3}{X_3}\right],
\end{aligned}
$$

$$y_{6m}=y_0\prod_{j=1}^{m}v_{6j}w_{6j-1}u_{6j-2}v_{6j-3}w_{6j-4}u_{6j-5}$$

$$=y_0\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{(X_2v_0-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_0-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_5-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_5-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right] \quad (3.1.111)$$
$$\times\left[\frac{1}{X_1}\frac{(X_1u_4-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_4-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_3-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_3-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right]$$
$$\times\left[\frac{1}{X_3}\frac{(X_3w_2-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_2-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_1-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_1-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right],$$

$$z_{6m}=z_0\prod_{j=1}^{m}w_{6j}u_{6j-1}v_{6j-2}w_{6j-3}u_{6j-4}v_{6j-5}$$

$$=z_0\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{(X_3w_0-Y_3)(j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}}{(X_3w_0-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_5-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_5-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right] \quad (3.1.112)$$
$$\times\left[\frac{1}{X_2}\frac{(X_2v_4-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_4-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_3-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_3-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right]$$
$$\times\left[\frac{1}{X_1}\frac{(X_1u_2-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_2-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_1-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_1-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right]$$

olur. Denklem (3.1.85)'de $i=1$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$x_{6m+1}=x_1\prod_{j=1}^{m}u_{6j+1}v_{6j}w_{6j-1}u_{6j-2}v_{6j-3}w_{6j-4}$$

$$=x_1\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{(X_1u_1-Y_1)(j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}}{(X_1u_1-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_0-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_0-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right] \quad (3.1.113)$$
$$\times\left[\frac{1}{X_3}\frac{(X_3w_5-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_5-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_4-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_4-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right]$$
$$\times\left[\frac{1}{X_2}\frac{(X_2v_3-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_3-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_2-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_2-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right],$$

$$y_{6m+1}=y_1\prod_{j=1}^{m}v_{6j+1}w_{6j}u_{6j-1}v_{6j-2}w_{6j-3}u_{6j-4}$$

$$=y_1\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{(X_2v_1-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_1-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_0-Y_3)(j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}}{(X_3w_0-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right] \quad (3.1.114)$$
$$\times\left[\frac{1}{X_1}\frac{(X_1u_5-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_5-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_4-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_4-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right]$$
$$\times\left[\frac{1}{X_3}\frac{(X_3w_3-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_3-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_2-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_2-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right],$$

$$
\begin{aligned}
z_{6m+1} &= z_1\prod_{j=1}^{m} w_{6j+1}u_{6j}v_{6j-1}w_{6j-2}u_{6j-3}v_{6j-4} \\
&= z_1\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{(X_3w_1-Y_3)\left((j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}\right)}{(X_3w_1-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_0-Y_1)\left((j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}\right)}{(X_1u_0-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right] \\
&\times\left[\frac{1}{X_2}\frac{(X_2v_5-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_5-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_4-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_4-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right] \\
&\times\left[\frac{1}{X_1}\frac{(X_1u_3-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_3-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_2-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_2-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right]
\end{aligned}
\tag{3.1.115}
$$

elde edilir. Denklem (3.1.85)'de $i=2$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$
\begin{aligned}
x_{6m+2} &= x_2\prod_{j=1}^{m} u_{6j+2}v_{6j+1}w_{6j}u_{6j-1}v_{6j-2}w_{6j-3} \\
&= x_2\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{(X_1u_2-Y_1)\left((j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}\right)}{(X_1u_2-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_1-Y_2)\left((j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}\right)}{(X_2v_1-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right] \\
&\times\left[\frac{1}{X_3}\frac{(X_3w_0-Y_3)\left((j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}\right)}{(X_3w_0-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_5-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_5-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right] \\
&\times\left[\frac{1}{X_2}\frac{(X_2v_4-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_4-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_3-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_3-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right],
\end{aligned}
\tag{3.1.116}
$$

$$
\begin{aligned}
y_{6m+2} &= y_2\prod_{j=1}^{m} v_{6j+2}w_{6j+1}u_{6j}v_{6j-1}w_{6j-2}u_{6j-3}, \\
&= y_2\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{(X_2v_2-Y_2)\left((j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}\right)}{(X_2v_2-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_1-Y_3)\left((j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}\right)}{(X_3w_1-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right] \\
&\times\left[\frac{1}{X_1}\frac{(X_1u_0-Y_1)\left((j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}\right)}{(X_1u_0-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_5-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_5-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right] \\
&\times\left[\frac{1}{X_3}\frac{(X_3w_4-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_4-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_3-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_3-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right],
\end{aligned}
\tag{3.1.117}
$$

$$
\begin{aligned}
z_{6m+2} &= z_2\prod_{j=1}^{m} w_{6j+2}u_{6j+1}v_{6j}w_{6j-1}u_{6j-2}v_{6j-3} \\
&= z_2\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{(X_3w_2-Y_3)\left((j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}\right)}{(X_3w_2-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_1-Y_1)\left((j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}\right)}{(X_1u_1-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right] \\
&\times\left[\frac{1}{X_2}\frac{(X_2v_0-Y_2)\left((j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}\right)}{(X_2v_0-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_5-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_5-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right] \\
&\times\left[\frac{1}{X_1}\frac{(X_1u_4-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_4-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_3-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_3-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right]
\end{aligned}
\tag{3.1.118}
$$

olur. Denklem (3.1.85)'de $i=3$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$
\begin{aligned}
x_{6m+3} &= x_3\prod_{j=1}^{m} u_{6j+3}v_{6j+2}w_{6j+1}u_{6j}v_{6j-1}w_{6j-2} \\
&= x_3\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{(X_1u_3-Y_1)(j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}}{(X_1u_3-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_2-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_2-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right] \\
&\times\left[\frac{1}{X_3}\frac{(X_3w_1-Y_3)(j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}}{(X_3w_1-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_0-Y_1)(j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}}{(X_1u_0-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right] \\
&\times\left[\frac{1}{X_2}\frac{(X_2v_5-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_5-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_4-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_4-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right],
\end{aligned}
\tag{3.1.119}
$$

$$
\begin{aligned}
y_{6m+3} &= y_3\prod_{j=1}^{m} v_{6j+3}w_{6j+2}u_{6j+1}v_{6j}w_{6j-1}u_{6j-2} \\
&= y_3\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{(X_2v_3-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_3-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_2-Y_3)(j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}}{(X_3w_2-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right] \\
&\times\left[\frac{1}{X_1}\frac{(X_1u_1-Y_1)(j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}}{(X_1u_1-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_0-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_0-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right] \\
&\times\left[\frac{1}{X_3}\frac{(X_3w_5-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_5-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_4-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_4-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right],
\end{aligned}
\tag{3.1.120}
$$

$$
\begin{aligned}
z_{6m+3} &= z_3\prod_{j=1}^{m} w_{6j+3}u_{6j+2}v_{6j+1}w_{6j}u_{6j-1}v_{6j-2} \\
&= z_3\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{(X_3w_3-Y_3)(j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}}{(X_3w_3-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_2-Y_1)(j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}}{(X_1u_2-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right] \\
&\times\left[\frac{1}{X_2}\frac{(X_2v_1-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_1-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_0-Y_3)(j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}}{(X_3w_0-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right] \\
&\times\left[\frac{1}{X_1}\frac{(X_1u_5-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_5-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_4-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_4-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right]
\end{aligned}
\tag{3.1.121}
$$

olur. Denklem (3.1.85)'de $i=4$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$
\begin{aligned}
x_{6m+4} &= x_4\prod_{j=1}^{m} u_{6j+4}v_{6j+3}w_{6j+2}u_{6j+1}v_{6j}w_{6j-1} \\
&= x_4\prod_{j=1}^{m}\left[\frac{1}{X_1}\frac{(X_1u_4-Y_1)(j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}}{(X_1u_4-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_3-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_3-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right] \\
&\times\left[\frac{1}{X_3}\frac{(X_3w_2-Y_3)(j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}}{(X_3w_2-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_1-Y_1)(j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}}{(X_1u_1-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right] \\
&\times\left[\frac{1}{X_2}\frac{(X_2v_0-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_0-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_5-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}{(X_3w_5-Y_3)(j-1)\lambda_5^{\,j-2}-(j-2)\lambda_5^{\,j-1}}+\frac{Y_3}{X_3}\right],
\end{aligned}
\tag{3.1.122}
$$

$$y_{6m+4}=y_4\prod_{j=1}^{m} v_{6j+4}w_{6j+3}u_{6j+2}v_{6j+1}w_{6j}u_{6j-1}$$

$$=y_4\prod_{j=1}^{m}\left[\frac{1}{X_2}\frac{(X_2v_4-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_4-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_3-Y_3)(j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}}{(X_3w_3-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right]$$

$$\times\left[\frac{1}{X_1}\frac{(X_1u_2-Y_1)(j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}}{(X_1u_2-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_1-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_1-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right]$$

$$\times\left[\frac{1}{X_3}\frac{(X_3w_0-Y_3)(j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}}{(X_3w_0-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_5-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}{(X_1u_5-Y_1)(j-1)\lambda_1^{\,j-2}-(j-2)\lambda_1^{\,j-1}}+\frac{Y_1}{X_1}\right], \quad (3.1.123)$$

$$z_{6m+4}=z_4\prod_{j=1}^{m} w_{6j+4}u_{6j+3}v_{6j+2}w_{6j+1}u_{6j}v_{6j-1}$$

$$=z_4\prod_{j=1}^{m}\left[\frac{1}{X_3}\frac{(X_3w_4-Y_3)(j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}}{(X_3w_4-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right]\times\left[\frac{1}{X_1}\frac{(X_1u_3-Y_1)(j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}}{(X_1u_3-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right]$$

$$\times\left[\frac{1}{X_2}\frac{(X_2v_2-Y_2)(j+1)\lambda_3^{\,j}-j\lambda_3^{\,j+1}}{(X_2v_2-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}+\frac{Y_2}{X_2}\right]\times\left[\frac{1}{X_3}\frac{(X_3w_1-Y_3)(j+1)\lambda_5^{\,j}-j\lambda_5^{\,j+1}}{(X_3w_1-Y_3)\,j\lambda_5^{\,j-1}-(j-1)\lambda_5^{\,j}}+\frac{Y_3}{X_3}\right]$$

$$\times\left[\frac{1}{X_1}\frac{(X_1u_0-Y_1)(j+1)\lambda_1^{\,j}-j\lambda_1^{\,j+1}}{(X_1u_0-Y_1)\,j\lambda_1^{\,j-1}-(j-1)\lambda_1^{\,j}}+\frac{Y_1}{X_1}\right]\times\left[\frac{1}{X_2}\frac{(X_2v_5-Y_2)\,j\lambda_3^{\,j-1}-(j-1)\lambda_3^{\,j}}{(X_2v_5-Y_2)(j-1)\lambda_3^{\,j-2}-(j-2)\lambda_3^{\,j-1}}+\frac{Y_2}{X_2}\right] \quad (3.1.124)$$

elde edilir. $\lambda_1=\lambda_2,\ \lambda_3=\lambda_4\ ve\,\lambda_5=\lambda_6$ durumu için (3.1.107)-(3.1.124) çözümleri dikkate alındığında Sistem (3.1.34)' ün çözümlerini daha kısa formda yazılacak olursa, bazı basit hesaplamalar sonrası Sistem (3.1.34)' ün, $m\in\mathbb{N}_0\ ve\ \ i\in\{-1,0,1,2,3,4\}$ için, çözümü

$$
\begin{aligned}
x_{6m+i} = x_i \prod_{j=1}^{m} & \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_1\right)\left(j+1+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_1^{j+1+\left\lfloor \frac{i}{6} \right\rfloor}}{\left(X_1 u_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_1\right)\left(j+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_1^{j-1+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
\times & \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_2\right)\left(j+1+\left\lfloor \frac{i-1}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-1}{6} \right\rfloor\right)\lambda_3^{j+1+\left\lfloor \frac{i-1}{6} \right\rfloor}}{\left(X_2 v_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_2\right)\left(j+\left\lfloor \frac{i-1}{6} \right\rfloor\right)\lambda_3^{j-1+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-1}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i-1}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
\times & \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_5^{j+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_5^{j+1+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left(X_3 w_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_5^{j-1+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_5^{j+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
\times & \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_1\right)\left(j+1+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_1^{j+1+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left(X_1 u_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_1\right)\left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_1^{j-1+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
\times & \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_2\right)\left(j+1+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_3^{j+1+\left\lfloor \frac{i-4}{6} \right\rfloor}}{\left(X_2 v_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_2\right)\left(j+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_3^{j-1+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i-4}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \qquad (3.1.125) \\
\times & \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_5^{j+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_5^{j+1+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left(X_3 w_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_5^{j-1+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_5^{j+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right),
\end{aligned}
$$

$$
\begin{aligned}
y_{6m+i} = y_i \prod_{j=1}^{m} & \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_2\right)\left(j+1+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{j+1+\left\lfloor \frac{i}{6} \right\rfloor}}{\left(X_2 v_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_2\right)\left(j+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{j-1+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
\times & \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i-1}{6} \right\rfloor\right)\lambda_5^{j+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-1}{6} \right\rfloor\right)\lambda_5^{j+1+\left\lfloor \frac{i-1}{6} \right\rfloor}}{\left(X_3 w_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i-1}{6} \right\rfloor\right)\lambda_5^{j-1+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left(j-1+\frac{i-1}{6}\right)\lambda_5^{j+\left\lfloor \frac{i-1}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
\times & \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_1\right)\left(j+1+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_1^{j+1+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left(X_1 u_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_1\right)\left(j+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_1^{j-1+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
\times & \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_2\right)\left(j+1+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{j+1+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left(X_2 v_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_2\right)\left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{j-1+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
\times & \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_5^{j+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_5^{j+1+\left\lfloor \frac{i-4}{6} \right\rfloor}}{\left(X_3 w_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_5^{j-1+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_5^{j+\left\lfloor \frac{i-4}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \quad (3.1.126) \\
\times & \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_1\right)\left(j+1+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_1^{j+1+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left(X_1 u_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_1\right)\left(j+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_1^{j-1+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right),
\end{aligned}
$$

$$
\begin{aligned}
z_{6m+i} = z_i \prod_{j=1}^{m} & \left( \frac{1}{X_3} \frac{\left( X_3 w_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_3 \right)\left( j+1+\left\lfloor \frac{i}{6} \right\rfloor \right)\lambda_5^{\,j+\frac{i}{6}} - \left( j+\left\lfloor \frac{i}{6} \right\rfloor \right)\lambda_5^{\,j+1+\frac{i}{6}}}{\left( X_3 w_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_3 \right)\left( j+\left\lfloor \frac{i}{6} \right\rfloor \right)\lambda_5^{\,j-1+\frac{i}{6}} - \left( j-1+\left\lfloor \frac{i}{6} \right\rfloor \right)\lambda_5^{\,j+\frac{i}{6}}} + \frac{Y_3}{X_3} \right) \\
\times & \left( \frac{1}{X_1} \frac{\left( X_1 u_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_1 \right)\left( j+1+\left\lfloor \frac{i-1}{6} \right\rfloor \right)\lambda_1^{\,j+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-1}{6} \right\rfloor \right)\lambda_1^{\,j+1+\left\lfloor \frac{i-1}{6} \right\rfloor}}{\left( X_1 u_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_1 \right)\left( j+\left\lfloor \frac{i-1}{6} \right\rfloor \right)\lambda_1^{\,j-1+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-1}{6} \right\rfloor \right)\lambda_1^{\,j+\left\lfloor \frac{i-1}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
\times & \left( \frac{1}{X_2} \frac{\left( X_2 v_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_2 \right)\left( j+1+\left\lfloor \frac{i-2}{6} \right\rfloor \right)\lambda_3^{\,j+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-2}{6} \right\rfloor \right)\lambda_3^{\,j+1+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left( X_2 v_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_2 \right)\left( j+\left\lfloor \frac{i-2}{6} \right\rfloor \right)\lambda_3^{\,j-1+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-2}{6} \right\rfloor \right)\lambda_3^{\,j+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
\times & \left( \frac{1}{X_3} \frac{\left( X_3 w_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_3 \right)\left( j+1+\left\lfloor \frac{i-3}{6} \right\rfloor \right)\lambda_5^{\,j+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-3}{6} \right\rfloor \right)\lambda_5^{\,j+1+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left( X_3 w_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_3 \right)\left( j+\left\lfloor \frac{i-3}{6} \right\rfloor \right)\lambda_5^{\,j-1+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-3}{6} \right\rfloor \right)\lambda_5^{\,j+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
\times & \left( \frac{1}{X_1} \frac{\left( X_1 u_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_1 \right)\left( j+1+\left\lfloor \frac{i-4}{6} \right\rfloor \right)\lambda_1^{\,j+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-4}{6} \right\rfloor \right)\lambda_1^{\,j+1+\left\lfloor \frac{i-4}{6} \right\rfloor}}{\left( X_1 u_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_1 \right)\left( j+\left\lfloor \frac{i-4}{6} \right\rfloor \right)\lambda_1^{\,j-1+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-4}{6} \right\rfloor \right)\lambda_1^{\,j+\left\lfloor \frac{i-4}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \qquad (3.1.127) \\
\times & \left( \frac{1}{X_2} \frac{\left( X_2 v_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_2 \right)\left( j+1+\left\lfloor \frac{i-5}{6} \right\rfloor \right)\lambda_3^{\,j+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-5}{6} \right\rfloor \right)\lambda_3^{\,j+1+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left( X_2 v_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_2 \right)\left( j+\left\lfloor \frac{i-5}{6} \right\rfloor \right)\lambda_3^{\,j-1+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-5}{6} \right\rfloor \right)\lambda_3^{\,j+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right)
\end{aligned}
$$

elde edilir. Burada $\lfloor x \rfloor$ literatürde iyi bilinen tam değeri ifade eder.

**3. durum** $\lambda_1=\lambda_2,\ \lambda_3\neq\lambda_4,\ \lambda_5\neq\lambda_6$ **ise**

Denklem (3.1.85)'de $i\in\{-1,0,1,2,3,4\}$ ve $m\in\mathbb{N}_0$ için $x_{6m+i},y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$\begin{aligned}
x_{6m+i}=x_i\prod_{j=1}^{m}&\left(\frac{1}{X_1}\frac{\left(X_1u_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_1\right)\left(j+1+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_1^{j+1+\left\lfloor\frac{i}{6}\right\rfloor}}{\left(X_1u_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_1\right)\left(j+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_1^{j-1+\left\lfloor\frac{i}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)\\
&\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_2\hat{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}}{\left(X_2v_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_3w_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)\\
&\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_1\right)\left(j+1+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i-3}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_1^{j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}}{\left(X_1u_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_1\right)\left(j+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_1^{j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i-3}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)\\
&\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_2\hat{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}}{\left(X_2v_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_3w_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right),
\end{aligned}\tag{3.1.128}$$

$$\begin{aligned}
y_{6m+i}=y_i\prod_{j=1}^{m}&\left(\frac{1}{X_2}\frac{\left(X_2v_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i}{6}\right\rfloor}-Z_2\hat{s}_{j+\left\lfloor\frac{i}{6}\right\rfloor}}{\left(X_2v_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\left\lfloor\frac{i}{6}\right\rfloor}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}}{\left(X_3w_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)\\
&\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_1\right)\left(j+1+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i-2}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_1^{j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_1u_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_1\right)\left(j+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_1^{j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)\\
&\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_2\hat{s}_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}}{\left(X_2v_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\frac{i-3}{6}}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}}{\left(X_3w_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)\\
&\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_1\right)\left(j+1+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i-5}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_1^{j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_1u_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_1\right)\left(j+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_1^{j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right),
\end{aligned}\tag{3.1.129}$$

$$z_{6m+i} = z_i \prod_{j=1}^{m} \left( \frac{1}{X_3} \frac{\left( X_3 w_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_3 \right) \tilde{s}_{j+1+\left\lfloor \frac{i}{6} \right\rfloor} - Z_3 \tilde{s}_{j+\left\lfloor \frac{i}{6} \right\rfloor}}{\left( X_3 w_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_3 \right) \tilde{s}_{j+\left\lfloor \frac{i}{6} \right\rfloor} - Z_3 \tilde{s}_{j-1+\left\lfloor \frac{i}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right)$$

$$\times \left( \frac{1}{X_1} \frac{\left( X_1 u_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_1 \right)\left( \left( j+1+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_1^{j+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_1^{j+1+\left\lfloor \frac{i-1}{6} \right\rfloor} \right)}{\left( X_1 u_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_1 \right)\left( \left( j+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_1^{j-1+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_1^{j+\left\lfloor \frac{i-1}{6} \right\rfloor} \right)} + \frac{Y_1}{X_1} \right)$$

$$\times \left( \frac{1}{X_2} \frac{\left( X_2 v_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_2 \right) \hat{s}_{j+1+\left\lfloor \frac{i-2}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left( X_2 v_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_2 \right) \hat{s}_{j+\left\lfloor \frac{i-2}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \times \left( \frac{1}{X_3} \frac{\left( X_3 w_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_3 \right) \tilde{s}_{j+1+\left\lfloor \frac{i-3}{6} \right\rfloor} - Z_3 \tilde{s}_{j+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left( X_3 w_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_3 \right) \tilde{s}_{j+\left\lfloor \frac{i-3}{6} \right\rfloor} - Z_3 \tilde{s}_{j-1+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right)$$

$$\times \left( \frac{1}{X_1} \frac{\left( X_1 u_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_1 \right)\left( \left( j+1+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_1^{j+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_1^{j+1+\left\lfloor \frac{i-4}{6} \right\rfloor} \right)}{\left( X_1 u_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_1 \right)\left( \left( j+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_1^{j-1+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_1^{j+\left\lfloor \frac{i-4}{6} \right\rfloor} \right)} + \frac{Y_1}{X_1} \right)$$

$$\times \left( \frac{1}{X_2} \frac{\left( X_2 v_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_2 \right) \hat{s}_{j+1+\left\lfloor \frac{i-5}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left( X_2 v_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_2 \right) \hat{s}_{j+\left\lfloor \frac{i-5}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \qquad (3.1.130)$$

olarak bulunur. Burada $\lfloor x \rfloor$ literatürde iyi bilinen tam değeri ifade eder.

**4.durum $\lambda_1 \neq \lambda_2$, $\lambda_3 = \lambda_4$, $\lambda_5 \neq \lambda_6$ ise**

Denklem (3.1.85)'de $i \in \{-1,0,1,2,3,4\}$ ve $m \in \mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$x_{6m+i}=x_i\prod_{j=1}^{m}\left(\frac{1}{X_1}\frac{\left(X_1u_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i}{6}\right\rfloor}}{\left(X_1u_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)$$

$$\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_2\right)\left(\left(j+1+\left\lfloor\frac{i-1}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-1}{6}\right\rfloor}-\left(j+\frac{i-1}{6}\right)\lambda_3^{j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}\right)}{\left(X_2v_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_2\right)\left(\left(j+\left\lfloor\frac{i-1}{6}\right\rfloor\right)\lambda_3^{j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}-\left(j-1+\frac{i-1}{6}\right)\lambda_3^{j+\left\lfloor\frac{i-1}{6}\right\rfloor}\right)}+\frac{Y_2}{X_2}\right)$$

$$\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_3w_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}}{\left(X_1u_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)$$

$$\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_2\right)\left(\left(j+1+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-4}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_3^{j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}\right)}{\left(X_2v_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_2\right)\left(\left(j+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_3^{j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-4}{6}\right\rfloor}\right)}+\frac{Y_2}{X_2}\right)\qquad(3.1.131)$$

$$\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_3w_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right),$$

$$y_{6m+i}=y_i\prod_{j=1}^{m}\left(\frac{1}{X_2}\frac{\left(X_2v_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_2\right)\left(\left(j+1+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_3^{j+1+\left\lfloor\frac{i}{6}\right\rfloor}\right)}{\left(X_2v_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_2\right)\left(\left(j+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_3^{j-1+\left\lfloor\frac{i}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i}{6}\right\rfloor}\right)}+\frac{Y_2}{X_2}\right)$$

$$\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}}{\left(X_3w_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_1u_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)$$

$$\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_2\right)\left(\left(j+1+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-3}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_3^{j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}\right)}{\left(X_2v_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_2\right)\left(\left(j+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_3^{j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-3}{6}\right\rfloor}\right)}+\frac{Y_2}{X_2}\right)$$

$$\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}}{\left(X_3w_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_1u_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right),\ (3.1.132)$$

$$z_{6m+i}=z_i\prod_{j=1}^{m}\left(\frac{1}{X_3}\frac{\left(X_3w_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i}{6}\right\rfloor}}{\left(X_3w_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}}{\left(X_1u_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)$$

$$\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_2\right)\left(j+1+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{\,j+\left\lfloor\frac{i-2}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{\,j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_2v_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_2\right)\left(j+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{\,j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{\,j+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)$$

$$\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}}{\left(X_3w_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}}{\left(X_1u_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)$$

$$\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_2\right)\left(j+1+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{\,j+\left\lfloor\frac{i-5}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{\,j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_2v_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_2\right)\left(j+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{\,j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{\,j+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right) \quad (3.1.133)$$

elde edilir. Burada $\lfloor x\rfloor$ literatürde iyi bilinen tam değeri ifade eder.

**5. durum $\lambda_1\neq\lambda_2,\ \lambda_3\neq\lambda_4,\ \lambda_5=\lambda_6$ ise**

Denklem (3.1.85)'de $i\in\{-1,0,1,2,3,4\}$ ve $m\in\mathbb{N}_0$ için $x_{6m+i},y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$x_{6m+i}=x_i\prod_{j=1}^{m}\left(\frac{1}{X_1}\frac{\left(X_1u_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i}{6}\right\rfloor}}{\left(X_1u_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_2\hat{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}}{\left(X_2v_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)$$

$$\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_3\right)\left(j+1+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_5^{\,j+\left\lfloor\frac{i-2}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_5^{\,j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_3w_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_3\right)\left(j+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_5^{\,j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_5^{\,j+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)$$

$$\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}}{\left(X_1u_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_2\hat{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}}{\left(X_2v_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_2\right)\hat{s}_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_2\hat{s}_{j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)$$

$$\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_3\right)\left(j+1+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_5^{\,j+\left\lfloor\frac{i-5}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_5^{\,j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_3w_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_3\right)\left(j+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_5^{\,j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_5^{\,j+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right), \quad (3.1.134)$$

$$
\begin{aligned}
y_{6m+i} = y_i \prod_{j=1}^{m} & \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_2\right) \hat{s}_{j+1+\left\lfloor \frac{i}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i}{6} \right\rfloor}}{\left(X_2 v_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_2\right) \hat{s}_{j+\left\lfloor \frac{i}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
& \times \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i-1}{6} \right\rfloor\right) \lambda_5^{\,j+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-1}{6} \right\rfloor\right) \lambda_5^{\,j+1+\left\lfloor \frac{i-1}{6} \right\rfloor}}{\left(X_3 w_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i-1}{6} \right\rfloor\right) \lambda_5^{\,j-1+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-1}{6} \right\rfloor\right) \lambda_5^{\,j+\left\lfloor \frac{i-1}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
& \times \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_1\right) s_{j+1+\left\lfloor \frac{i-2}{6} \right\rfloor} - Z_1 s_{j+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left(X_1 u_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_1\right) s_{j+\left\lfloor \frac{i-2}{6} \right\rfloor} - Z_1 s_{j-1+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \times \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_2\right) \hat{s}_{j+1+\left\lfloor \frac{i-3}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left(X_2 v_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_2\right) \hat{s}_{j+\left\lfloor \frac{i-3}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
& \times \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i-4}{6} \right\rfloor\right) \lambda_5^{\,j+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-4}{6} \right\rfloor\right) \lambda_5^{\,j+1+\left\lfloor \frac{i-4}{6} \right\rfloor}}{\left(X_3 w_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i-4}{6} \right\rfloor\right) \lambda_5^{\,j-1+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-4}{6} \right\rfloor\right) \lambda_5^{\,j+\left\lfloor \frac{i-4}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
& \times \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_1\right) s_{j+1+\left\lfloor \frac{i-5}{6} \right\rfloor} - Z_1 s_{j+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left(X_1 u_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_1\right) s_{j+\left\lfloor \frac{i-5}{6} \right\rfloor} - Z_1 s_{j-1+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right),
\end{aligned}
\tag{3.1.135}
$$

$$
\begin{aligned}
z_{6m+i} = z_i \prod_{j=1}^{m} & \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i}{6} \right\rfloor\right) \lambda_5^{\,j+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i}{6} \right\rfloor\right) \lambda_5^{\,j+1+\left\lfloor \frac{i}{6} \right\rfloor}}{\left(X_3 w_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i}{6} \right\rfloor\right) \lambda_5^{\,j-1+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i}{6} \right\rfloor\right) \lambda_5^{\,j+\left\lfloor \frac{i}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
& \times \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_1\right) s_{j+1+\left\lfloor \frac{i-1}{6} \right\rfloor} - Z_1 s_{j+\left\lfloor \frac{i-1}{6} \right\rfloor}}{\left(X_1 u_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_1\right) s_{j+\left\lfloor \frac{i-1}{6} \right\rfloor} - Z_1 s_{j-1+\left\lfloor \frac{i-1}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \times \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_2\right) \hat{s}_{j+1+\left\lfloor \frac{i-2}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left(X_2 v_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_2\right) \hat{s}_{j+\left\lfloor \frac{i-2}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
& \times \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i-3}{6} \right\rfloor\right) \lambda_5^{\,j+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right) \lambda_5^{\,j+1+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left(X_3 w_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right) \lambda_5^{\,j-1+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-3}{6} \right\rfloor\right) \lambda_5^{\,j+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
& \times \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_1\right) s_{j+1+\left\lfloor \frac{i-4}{6} \right\rfloor} - Z_1 s_{j+\left\lfloor \frac{i-4}{6} \right\rfloor}}{\left(X_1 u_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_1\right) s_{j+\left\lfloor \frac{i-4}{6} \right\rfloor} - Z_1 s_{j-1+\left\lfloor \frac{i-4}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \times \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_2\right) \hat{s}_{j+1+\left\lfloor \frac{i-5}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left(X_2 v_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_2\right) \hat{s}_{j+\left\lfloor \frac{i-5}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right)
\end{aligned}
\tag{3.1.136}
$$

elde edilir. Burada $\lfloor x \rfloor$ literatürde iyi bilinen tam değeri ifade eder.

**6. durum $\lambda_1=\lambda_2,\ \lambda_3=\lambda_4,\ \lambda_5\neq\lambda_6$ ise**

Denklem (3.1.85)'de $i\in\{-1,0,1,2,3,4\}$ ve $m\in\mathbb{N}_0$ için $x_{6m+i},y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$
\begin{aligned}
x_{6m+i}=x_i\prod_{j=1}^{m}&\left(\frac{1}{X_1}\frac{\left(X_1u_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_1\right)\left(j+1+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_1^{\,j+\left\lfloor\frac{i}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_1^{\,j+1+\left\lfloor\frac{i}{6}\right\rfloor}}{\left(X_1u_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_1\right)\left(j+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_1^{\,j-1+\left\lfloor\frac{i}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_1^{\,j+\left\lfloor\frac{i}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)\\
&\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_2\right)\left(j+1+\left\lfloor\frac{i-1}{6}\right\rfloor\right)\lambda_3^{\,j+\left\lfloor\frac{i-1}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-1}{6}\right\rfloor\right)\lambda_3^{\,j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}}{\left(X_2v_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_2\right)\left(j+\left\lfloor\frac{i-1}{6}\right\rfloor\right)\lambda_3^{\,j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-1}{6}\right\rfloor\right)\lambda_3^{\,j+\left\lfloor\frac{i-1}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)\\
&\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_3w_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-2}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)\\
&\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_1\right)\left(j+1+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_1^{\,j+\left\lfloor\frac{i-3}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_1^{\,j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}}{\left(X_1u_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_1\right)\left(j+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_1^{\,j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_1^{\,j+\left\lfloor\frac{i-3}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)\\
&\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_2\right)\left(j+1+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_3^{\,j+\left\lfloor\frac{i-4}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_3^{\,j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}}{\left(X_2v_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_2\right)\left(j+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_3^{\,j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_3^{\,j+\left\lfloor\frac{i-4}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)\quad(3.1.137)\\
&\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_3w_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-5}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right),
\end{aligned}
$$

$$
\begin{aligned}
y_{6m+i} = y_i \prod_{j=1}^{m} & \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_2\right)\left(j+1+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{j+1+\left\lfloor \frac{i}{6} \right\rfloor}}{\left(X_2 v_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_2\right)\left(j+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{j-1+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
& \times \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_3\right)\tilde{s}_{j+1+\left\lfloor \frac{i-1}{6} \right\rfloor} - Z_3 \tilde{s}_{j+\left\lfloor \frac{i-1}{6} \right\rfloor}}{\left(X_3 w_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_3\right)\tilde{s}_{j+\left\lfloor \frac{i-1}{6} \right\rfloor} - Z_3 \tilde{s}_{j-1+\left\lfloor \frac{i-1}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
& \times \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_1\right)\left(j+1+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_1^{j+1+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left(X_1 u_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_1\right)\left(j+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_1^{j-1+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
& \times \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_2\right)\left(j+1+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{j+1+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left(X_2 v_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_2\right)\left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{j-1+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{j+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
& \times \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_3\right)\tilde{s}_{j+1+\left\lfloor \frac{i-4}{6} \right\rfloor} - Z_3 \tilde{s}_{j+\left\lfloor \frac{i-4}{6} \right\rfloor}}{\left(X_3 w_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_3\right)\tilde{s}_{j+\left\lfloor \frac{i-4}{6} \right\rfloor} - Z_3 \tilde{s}_{j-1+\left\lfloor \frac{i-4}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \qquad (3.1.138) \\
& \times \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_1\right)\left(j+1+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_1^{j+1+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left(X_1 u_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_1\right)\left(j+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_1^{j-1+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right)
\end{aligned}
$$

$$z_{6m+i}=z_i\prod_{j=1}^{m}\left(\frac{1}{X_3}\frac{\left(X_3w_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i}{6}\right\rfloor}}{\left(X_3w_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)$$

$$\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_1\right)\left(j+1+\left\lfloor\frac{i-1}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i-1}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-1}{6}\right\rfloor\right)\lambda_1^{j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}}{\left(X_1u_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_1\right)\left(j+\left\lfloor\frac{i-1}{6}\right\rfloor\right)\lambda_1^{j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-1}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i-1}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)$$

$$\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_2\right)\left(j+1+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-2}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_2v_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_2\right)\left(j+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)$$

$$\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_3\tilde{s}_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}}{\left(X_3w_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_3\right)\tilde{s}_{j+\left\lfloor\frac{i-3}{6}\right\rfloor}-Z_3\tilde{s}_{j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)$$

$$\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_1\right)\left(j+1+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i-4}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_1^{j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}}{\left(X_1u_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_1\right)\left(j+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_1^{j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-4}{6}\right\rfloor\right)\lambda_1^{j+\left\lfloor\frac{i-4}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)\qquad(3.1.139)$$

$$\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_2\right)\left(j+1+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-5}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_2v_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_2\right)\left(j+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)$$

elde edilir. Burada $\lfloor x\rfloor$ literatürde iyi bilinen tam değeri ifade eder.

**7. durum $\lambda_1\neq\lambda_2$, $\lambda_3=\lambda_4$, $\lambda_5=\lambda_6$ ise**

Denklem (3.1.85)'de $i\in\{-1,0,1,2,3,4\}$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$
\begin{aligned}
x_{6m+i} = x_i \prod_{j=1}^{m} & \left( \frac{1}{X_1} \frac{\left( X_1 u_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_1 \right) s_{j+1+\left\lfloor \frac{i}{6} \right\rfloor} - Z_1 s_{j+\left\lfloor \frac{i}{6} \right\rfloor}}{\left( X_1 u_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_1 \right) s_{j+\left\lfloor \frac{i}{6} \right\rfloor} - Z_1 s_{j-1+\left\lfloor \frac{i}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
& \times \left( \frac{1}{X_2} \frac{\left( X_2 v_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_2 \right) \left( j+1+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_3^{j+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_3^{j+1+\left\lfloor \frac{i-1}{6} \right\rfloor}}{\left( X_2 v_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_2 \right) \left( j+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_3^{j-1+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_3^{j+\left\lfloor \frac{i-1}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
& \times \left( \frac{1}{X_3} \frac{\left( X_3 w_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_3 \right) \left( j+1+\left\lfloor \frac{i-2}{6} \right\rfloor \right) \lambda_5^{j+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-2}{6} \right\rfloor \right) \lambda_5^{j+1+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left( X_3 w_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_3 \right) \left( j+\left\lfloor \frac{i-2}{6} \right\rfloor \right) \lambda_5^{j-1+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-2}{6} \right\rfloor \right) \lambda_5^{j+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
& \times \left( \frac{1}{X_1} \frac{\left( X_1 u_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_1 \right) s_{j+1+\left\lfloor \frac{i-3}{6} \right\rfloor} - Z_1 s_{j+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left( X_1 u_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_1 \right) s_{j+\left\lfloor \frac{i-3}{6} \right\rfloor} - Z_1 s_{j-1+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
& \times \left( \frac{1}{X_2} \frac{\left( X_2 v_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_2 \right) \left( j+1+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_3^{j+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_3^{j+1+\left\lfloor \frac{i-4}{6} \right\rfloor}}{\left( X_2 v_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_2 \right) \left( j+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_3^{j-1+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_3^{j+\left\lfloor \frac{i-4}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \quad (3.1.140) \\
& \times \left( \frac{1}{X_3} \frac{\left( X_3 w_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_3 \right) \left( j+1+\left\lfloor \frac{i-5}{6} \right\rfloor \right) \lambda_5^{j+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-5}{6} \right\rfloor \right) \lambda_5^{j+1+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left( X_3 w_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_3 \right) \left( j+\left\lfloor \frac{i-5}{6} \right\rfloor \right) \lambda_5^{j-1+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-5}{6} \right\rfloor \right) \lambda_5^{j+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right),
\end{aligned}
$$

$$
\begin{aligned}
y_{6m+i} = y_i \prod_{j=1}^{m} & \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_2\right)\left(j+1+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{\,j+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{\,j+1+\left\lfloor \frac{i}{6} \right\rfloor}}{\left(X_2 v_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_2\right)\left(j+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{\,j-1+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_3^{\,j+\left\lfloor \frac{i}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
& \times \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i-1}{6} \right\rfloor\right)\lambda_5^{\,j+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-1}{6} \right\rfloor\right)\lambda_5^{\,j+1+\left\lfloor \frac{i-1}{6} \right\rfloor}}{\left(X_3 w_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i-1}{6} \right\rfloor\right)\lambda_5^{\,j-1+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-1}{6} \right\rfloor\right)\lambda_5^{\,j+\left\lfloor \frac{i-1}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
& \times \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_1\right) s_{j+1+\left\lfloor \frac{i-2}{6} \right\rfloor} - Z_1 s_{j+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left(X_1 u_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_1\right) s_{j+\left\lfloor \frac{i-2}{6} \right\rfloor} - Z_1 s_{j-1+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
& \times \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_2\right)\left(j+1+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{\,j+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{\,j+1+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left(X_2 v_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_2\right)\left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{\,j-1+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_3^{\,j+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
& \times \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_5^{\,j+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_5^{\,j+1+\left\lfloor \frac{i-4}{6} \right\rfloor}}{\left(X_3 w_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_5^{\,j-1+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-4}{6} \right\rfloor\right)\lambda_5^{\,j+\left\lfloor \frac{i-4}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \quad (3.1.141) \\
& \times \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_1\right) s_{j+1+\left\lfloor \frac{i-5}{6} \right\rfloor} - Z_1 s_{j+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left(X_1 u_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_1\right) s_{j+\left\lfloor \frac{i-5}{6} \right\rfloor} - Z_1 s_{j-1+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right),
\end{aligned}
$$

$$z_{6m+i}=z_i\prod_{j=1}^{m}\left(\frac{1}{X_3}\frac{\left(X_3w_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_3\right)\left(j+1+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_5^{j+\left\lfloor\frac{i}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_5^{j+1+\left\lfloor\frac{i}{6}\right\rfloor}}{\left(X_3w_{i-6\left\lfloor\frac{i}{6}\right\rfloor}-Y_3\right)\left(j+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_5^{j-1+\left\lfloor\frac{i}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i}{6}\right\rfloor\right)\lambda_5^{j+\left\lfloor\frac{i}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)$$

$$\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}}{\left(X_1u_{i-1-6\left\lfloor\frac{i-1}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-1}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-1}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right)$$

$$\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_2\right)\left(j+1+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-2}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{j+1+\left\lfloor\frac{i-2}{6}\right\rfloor}}{\left(X_2v_{i-2-6\left\lfloor\frac{i-2}{6}\right\rfloor}-Y_2\right)\left(j+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{j-1+\left\lfloor\frac{i-2}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-2}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-2}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)$$

$$\times\left(\frac{1}{X_3}\frac{\left(X_3w_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_3\right)\left(j+1+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_5^{j+\left\lfloor\frac{i-3}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_5^{j+1+\left\lfloor\frac{i-3}{6}\right\rfloor}}{\left(X_3w_{i-3-6\left\lfloor\frac{i-3}{6}\right\rfloor}-Y_3\right)\left(j+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_5^{j-1+\left\lfloor\frac{i-3}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-3}{6}\right\rfloor\right)\lambda_5^{j+\left\lfloor\frac{i-3}{6}\right\rfloor}}+\frac{Y_3}{X_3}\right)$$

$$\times\left(\frac{1}{X_1}\frac{\left(X_1u_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_1\right)s_{j+1+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_1s_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}}{\left(X_1u_{i-4-6\left\lfloor\frac{i-4}{6}\right\rfloor}-Y_1\right)s_{j+\left\lfloor\frac{i-4}{6}\right\rfloor}-Z_1s_{j-1+\left\lfloor\frac{i-4}{6}\right\rfloor}}+\frac{Y_1}{X_1}\right) \qquad (3.1.142)$$

$$\times\left(\frac{1}{X_2}\frac{\left(X_2v_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_2\right)\left(j+1+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-5}{6}\right\rfloor}-\left(j+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{j+1+\left\lfloor\frac{i-5}{6}\right\rfloor}}{\left(X_2v_{i-5-6\left\lfloor\frac{i-5}{6}\right\rfloor}-Y_2\right)\left(j+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{j-1+\left\lfloor\frac{i-5}{6}\right\rfloor}-\left(j-1+\left\lfloor\frac{i-5}{6}\right\rfloor\right)\lambda_3^{j+\left\lfloor\frac{i-5}{6}\right\rfloor}}+\frac{Y_2}{X_2}\right)$$

elde edilir. Burada $\lfloor x\rfloor$ literatürde iyi bilinen tam değeri ifade eder.

**8.durum $\lambda_1=\lambda_2$, $\lambda_3\neq\lambda_4$, $\lambda_5=\lambda_6$ ise**

Denklem (3.1.85)'de $i\in\{-1,0,1,2,3,4\}$ ve $m\in\mathbb{N}_0$ için $x_{6m+i}, y_{6m+i}$ ve $z_{6m+i}$ çözümleri sırasıyla

$$
\begin{aligned}
x_{6m+i} = x_i \prod_{j=1}^{m} & \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_1\right)\left(j+1+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_1^{j+1+\left\lfloor \frac{i}{6} \right\rfloor}}{\left(X_1 u_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_1\right)\left(j+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_1^{j-1+\left\lfloor \frac{i}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
& \times \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_2\right)\hat{s}_{j+1+\left\lfloor \frac{i-1}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i-1}{6} \right\rfloor}}{\left(X_2 v_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_2\right)\hat{s}_{j+\left\lfloor \frac{i-1}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i-1}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
& \times \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_5^{j+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_5^{j+1+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left(X_3 w_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_5^{j-1+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-2}{6} \right\rfloor\right)\lambda_5^{j+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
& \times \left( \frac{1}{X_1} \frac{\left(X_1 u_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_1\right)\left(j+1+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_1^{j+1+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left(X_1 u_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_1\right)\left(j+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_1^{j-1+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-3}{6} \right\rfloor\right)\lambda_1^{j+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
& \times \left( \frac{1}{X_2} \frac{\left(X_2 v_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_2\right)\hat{s}_{j+1+\left\lfloor \frac{i-4}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i-4}{6} \right\rfloor}}{\left(X_2 v_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_2\right)\hat{s}_{j+\left\lfloor \frac{i-4}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i-4}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \qquad (3.1.143) \\
& \times \left( \frac{1}{X_3} \frac{\left(X_3 w_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_3\right)\left(j+1+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_5^{j+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left(j+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_5^{j+1+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left(X_3 w_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_3\right)\left(j+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_5^{j-1+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left(j-1+\left\lfloor \frac{i-5}{6} \right\rfloor\right)\lambda_5^{j+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right),
\end{aligned}
$$

$$
\begin{aligned}
y_{6m+i} = y_i \prod_{j=1}^{m} & \left( \frac{1}{X_2} \frac{\left( X_2 v_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_2 \right) \hat{s}_{j+1+\left\lfloor \frac{i}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i}{6} \right\rfloor}}{\left( X_2 v_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_2 \right) \hat{s}_{j+\left\lfloor \frac{i}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
\times & \left( \frac{1}{X_3} \frac{\left( X_3 w_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_3 \right) \left( j+1+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_5^{j+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_5^{j+1+\left\lfloor \frac{i-1}{6} \right\rfloor}}{\left( X_3 w_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_3 \right) \left( j+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_5^{j-1+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-1}{6} \right\rfloor \right) \lambda_5^{j+\left\lfloor \frac{i-1}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
\times & \left( \frac{1}{X_1} \frac{\left( X_1 u_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_1 \right) \left( j+1+\left| \frac{i-2}{6} \right\rangle \right) \lambda_1^{j+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-2}{6} \right\rfloor \right) \lambda_1^{j+1+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left( X_1 u_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_1 \right) \left( j+\left\lfloor \frac{i-2}{6} \right\rfloor \right) \lambda_1^{j-1+\left\lfloor \frac{i-2}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-2}{6} \right\rfloor \right) \lambda_1^{j+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
\times & \left( \frac{1}{X_2} \frac{\left( X_2 v_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_2 \right) \hat{s}_{j+1+\left\lfloor \frac{i-3}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left( X_2 v_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_2 \right) \hat{s}_{j+\left\lfloor \frac{i-3}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
\times & \left( \frac{1}{X_3} \frac{\left( X_3 w_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_3 \right) \left( j+1+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_5^{j+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_5^{j+1+\left\lfloor \frac{i-4}{6} \right\rfloor}}{\left( X_3 w_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_3 \right) \left( j+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_5^{j-1+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-4}{6} \right\rfloor \right) \lambda_5^{j+\left\lfloor \frac{i-4}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \quad (3.1.144) \\
\times & \left( \frac{1}{X_1} \frac{\left( X_1 u_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_1 \right) \left( j+1+\left\lfloor \frac{i-5}{6} \right\rfloor \right) \lambda_1^{j+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-5}{6} \right\rfloor \right) \lambda_1^{j+1+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left( X_1 u_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_1 \right) \left( j+\left\lfloor \frac{i-5}{6} \right\rfloor \right) \lambda_1^{j-1+\left\lfloor \frac{i-5}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-5}{6} \right\rfloor \right) \lambda_1^{j+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right),
\end{aligned}
$$

$$
\begin{aligned}
z_{6m+i} = z_i \prod_{j=1}^{m} & \left( \frac{1}{X_3} \frac{\left( X_3 w_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_3 \right)\left( j+1+\left\lfloor \frac{i}{6} \right\rfloor \right)\lambda_5^{j+\left\lfloor \frac{i}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i}{6} \right\rfloor \right)\lambda_5^{j+1+\left\lfloor \frac{i}{6} \right\rfloor}}{\left( X_3 w_{i-6\left\lfloor \frac{i}{6} \right\rfloor} - Y_3 \right)\left( j+\left\lfloor \frac{i}{6} \right\rfloor \right)\lambda_5^{j-1+\left\lfloor \frac{i}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i}{6} \right\rfloor \right)\lambda_5^{j+\left\lfloor \frac{i}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
& \times \left( \frac{1}{X_1} \frac{\left( X_1 u_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_1 \right)\left( j+1+\left\lfloor \frac{i-1}{6} \right\rfloor \right)\lambda_1^{j+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-1}{6} \right\rfloor \right)\lambda_1^{j+1+\left\lfloor \frac{i-1}{6} \right\rfloor}}{\left( X_1 u_{i-1-6\left\lfloor \frac{i-1}{6} \right\rfloor} - Y_1 \right)\left( j+\left\lfloor \frac{i-1}{6} \right\rfloor \right)\lambda_1^{j-1+\left\lfloor \frac{i-1}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-1}{6} \right\rfloor \right)\lambda_1^{j+\left\lfloor \frac{i-1}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \\
& \times \left( \frac{1}{X_2} \frac{\left( X_2 v_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_2 \right)\hat{s}_{j+1+\left\lfloor \frac{i-2}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i-2}{6} \right\rfloor}}{\left( X_2 v_{i-2-6\left\lfloor \frac{i-2}{6} \right\rfloor} - Y_2 \right)\hat{s}_{j+\left\lfloor \frac{i-2}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i-2}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right) \\
& \times \left( \frac{1}{X_3} \frac{\left( X_3 w_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_3 \right)\left( j+1+\left\lfloor \frac{i-3}{6} \right\rfloor \right)\lambda_5^{j+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-3}{6} \right\rfloor \right)\lambda_5^{j+1+\left\lfloor \frac{i-3}{6} \right\rfloor}}{\left( X_3 w_{i-3-6\left\lfloor \frac{i-3}{6} \right\rfloor} - Y_3 \right)\left( j+\left\lfloor \frac{i-3}{6} \right\rfloor \right)\lambda_5^{j-1+\left\lfloor \frac{i-3}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-3}{6} \right\rfloor \right)\lambda_5^{j+\left\lfloor \frac{i-3}{6} \right\rfloor}} + \frac{Y_3}{X_3} \right) \\
& \times \left( \frac{1}{X_1} \frac{\left( X_1 u_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_1 \right)\left( j+1+\left\lfloor \frac{i-4}{6} \right\rfloor \right)\lambda_1^{j+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left( j+\left\lfloor \frac{i-4}{6} \right\rfloor \right)\lambda_1^{j+1+\left\lfloor \frac{i-4}{6} \right\rfloor}}{\left( X_1 u_{i-4-6\left\lfloor \frac{i-4}{6} \right\rfloor} - Y_1 \right)\left( j+\left\lfloor \frac{i-4}{6} \right\rfloor \right)\lambda_1^{j-1+\left\lfloor \frac{i-4}{6} \right\rfloor} - \left( j-1+\left\lfloor \frac{i-4}{6} \right\rfloor \right)\lambda_1^{j+\left\lfloor \frac{i-4}{6} \right\rfloor}} + \frac{Y_1}{X_1} \right) \quad (3.1.145) \\
& \times \left( \frac{1}{X_2} \frac{\left( X_2 v_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_2 \right)\hat{s}_{j+1+\left\lfloor \frac{i-5}{6} \right\rfloor} - Z_2 \hat{s}_{j+\left\lfloor \frac{i-5}{6} \right\rfloor}}{\left( X_2 v_{i-5-6\left\lfloor \frac{i-5}{6} \right\rfloor} - Y_2 \right)\hat{s}_{j+\left\lfloor \frac{i-5}{6} \right\rfloor} - Z_2 \hat{s}_{j-1+\left\lfloor \frac{i-5}{6} \right\rfloor}} + \frac{Y_2}{X_2} \right)
\end{aligned}
$$

elde edilir. Burada $\lfloor x \rfloor$ literatürde iyi bilinen tam değeri ifade eder.

# 4. BÖLÜM

## 4.1. Sonuç ve Öneriler

Bu tez çalışmasında ilk olarak Sistem (3.1.1)'in katsayıların durumuna göre 10 farklı durumu incelenmiştir. Son durumda sistem kolayca çözülemeyeceğinden (3.1.35), (3.1.37), (3.1.39) sistemlerinde sırasıyla $\frac{x_n}{y_{n-1}} = u_n$, $\frac{y_n}{z_{n-1}} = v_n$, $\frac{z_n}{x_{n-1}} = w_n$, $n \in \mathbb{N}_0$, dönüşümü uygulanarak

(3.1.41) sistemi elde edilmiştir ve gerekli işlemler yapılarak iyi bilinen Riccati fark denklemine indirgenmiştir. Riccati fark denklemi dönüşümler yardımıyla 2. mertebeden sabit katsayılı lineer fark denklemine indirgenmiş ve karakteristik denklemin köklerinin durumlarına göre çözümler elde edilmiştir. Ardından $\frac{x_n}{y_{n-1}} = u_n$, $\frac{y_n}{z_{n-1}} = v_n$, $\frac{z_n}{x_{n-1}} = w_n$, $n \in \mathbb{N}_0$,

dönüşümünden Denklem (3.1.84) ve Denklem (3.1.85) elde edilmiştir. Fakat tüm durum boyunca karakteristik denkleminin köklerinin aynı veya farklı olması durumlarına göre 8 durumun oluştuğu görülmüş ve incelenmiştir. Elde edilen formüller kullanılarak Sistem (3.1.1)'in $\{(x_n, y_n, z_n)\}_{n\geq -1}$ çözümleri elde edilmiştir.

Buradan yola çıkarak (3.1.1) fark denklem sisteminin $k$- boyutlu fark denklem sistemi tanımlanarak çözümleri elde edilebilir ve sistemin çözümlerinin davranışları da incelenebilir.

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